NCERT Exemplar

Class 9 Maths

Chapter 8- Quadrilaterals

Exercise 8.1 (14 Multiple Choice Questions and Answers)

Question: 1

Three angles of a quadrilateral are 75º, 90º and 75º. The fourth angle is

(a) 90º

(b) 95º

(c) 105º

(d) 120º

Solution:

d

Let the fourth angle be 'x'.

Since the sum of all four angles of a quadrilateral is 360º, so

$$75^\circ + 90^\circ + 75^\circ + x^\circ = 360^\circ$$
$$\Rightarrow 240^\circ + x^\circ = 360^\circ$$
$$\Rightarrow x^\circ = 360^\circ - 240^\circ = 120^\circ$$

Hence, the fourth angle is 120º.

Question: 2

A diagonal of a rectangle is inclined to one side of the rectangle at 25°. The acute angle between the diagonals is:

(a) 55°

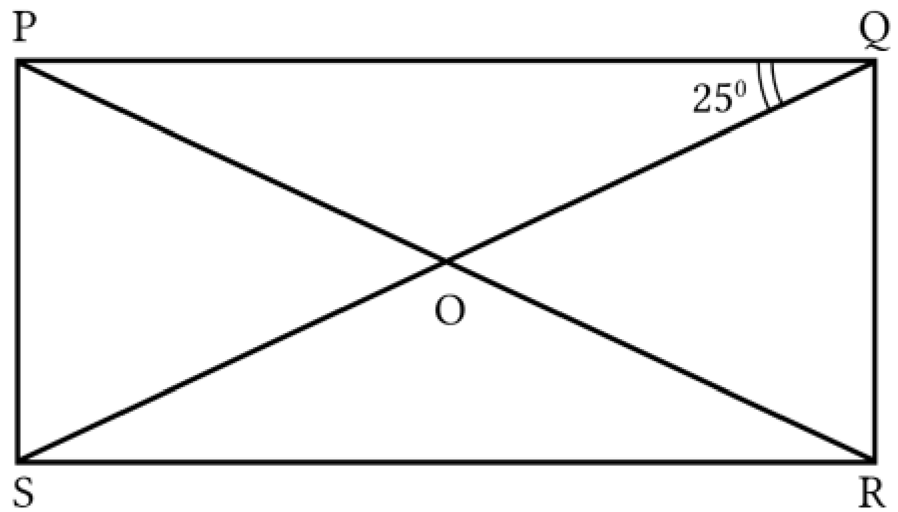
(b) 50°

(c) 40°

(d) 25°

Solution:

b



Let us consider a rectangle PQRS and diagonal SQ is inclined to PQ at 25°.

We need to find the acute angle between the diagonals.

Let both the diagonals intersect at point 'O'.

Since the diagonals of a rectangle bisect each other,

in ΔOPQ,

$\mathrm{OP} = \mathrm{OQ}$

$\Rightarrow \angle \mathrm{OQP} = \angle \mathrm{OPQ} = 25^{\circ}$

Since $\angle \mathrm{ROQ}$ is an exterior angle to $\Delta \mathrm{OPQ}$,

$\therefore \angle \mathrm{ROQ} = \angle \mathrm{OQP} + \angle \mathrm{OPQ} = 25^{\circ} + 25^{\circ} = 50^{\circ}$.

Also $\angle \mathrm{ROQ} + \angle \mathrm{QOP} = 180^{\circ}$ ($\because$ PR is a line segment),

$\Rightarrow \angle \mathrm{QOP} = 180^{\circ} - 50^{\circ} = 130^{\circ}$.

Hence, the acute angle between the diagonals is 50°.

Question: 3

ABCD is a rhombus such that $\angle \mathrm{ACB} = 40^{\circ}$.

Then $\angle \mathrm{ADB}$ is:

(a) 40°

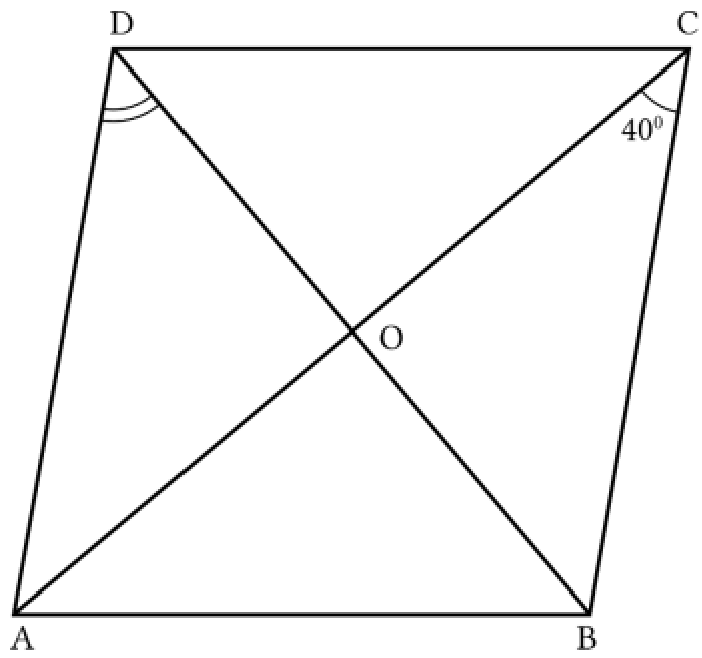
(b) 45°

(c) 50°

(d) 60º

Solution:

c



We are given that ABCD is a rhombus such that

$\angle ACB = 40^{o}$.

We need to find $\angle ADB$.

Since the diagonals of a rhombus are perpendicular

bisector of each other,

in ΔBOC, $\angle BOC = 90^{o}$.

Now,

$\angle BOC + \angle OCB + \angle OBC = 180^{o}$ (Sum of all angles in a triangle)

$\Rightarrow 90^{o} + 40^{o} + \angle OBC = 180^{o}$

$(\because \angle ACB = 40^{o} \Rightarrow \angle OCB = 40^{o})$

$\Rightarrow \angle OBC = 180^{o} - (90^{o} + 40^{o})$

$= 180^{o} - 130^{o} = 50^{o} \qquad \ldots.(I)$

Since $AB \parallel CD$,

$\Rightarrow \angle BDA = \angle DBC$ (Alternate interior angle)

$\Rightarrow \angle BDA = \angle OBC$

$\Rightarrow \angle BDA = 50^{o}$

Question: 4

The quadrilateral formed by joining the midpoints of the sides of a quadrilateral

PQRS, taken in order, is a rectangle, if:

(a) PQRS is a rectangle

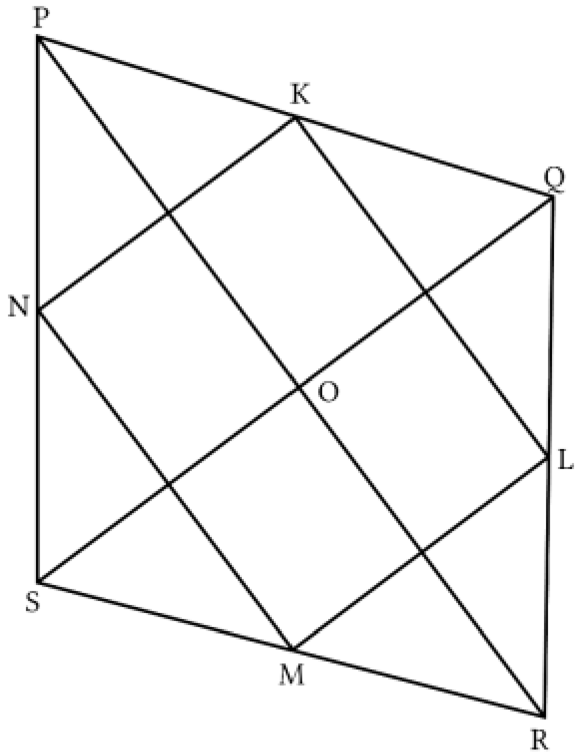
(b) PQRS is a parallelogram

(c) Diagonals of PQRS are perpendicular

(d) Diagonals of PQRS are equal

Solution:

c



Let KLMN be the quadrilateral formed by joining the midpoints of quadrilateral PQRS.

Now, if PQRS is a rectangle, then

diagonal PR = diagonal QS.

Then KLMN will be a square having all 4 sides equal.

If PQRS is a parallelogram, then the opposite sides will be parallel to each other and equal, but the angles may not be right angle.

If the diagonals of PQRS are equal, then all 4 sides of the quadrilateral KLMN will be equal.

So, option (a), (b) and (d) can't be the answer.

Now, if the diagonals of PQRS are perpendicular, then

by the midpoint theorem, the opposite sides are equal and parallel to each other.

Also, each angle of quadrilateral KLMN will be right angle.

So, KLMN will be a rectangle.

Hence, option(c) is the answer.

Question: 5

The quadrilateral formed by joining the midpoints of the sides of a quadrilateral

PQRS, taken in order, is a rhombus, if

(a) PQRS is a rhombus

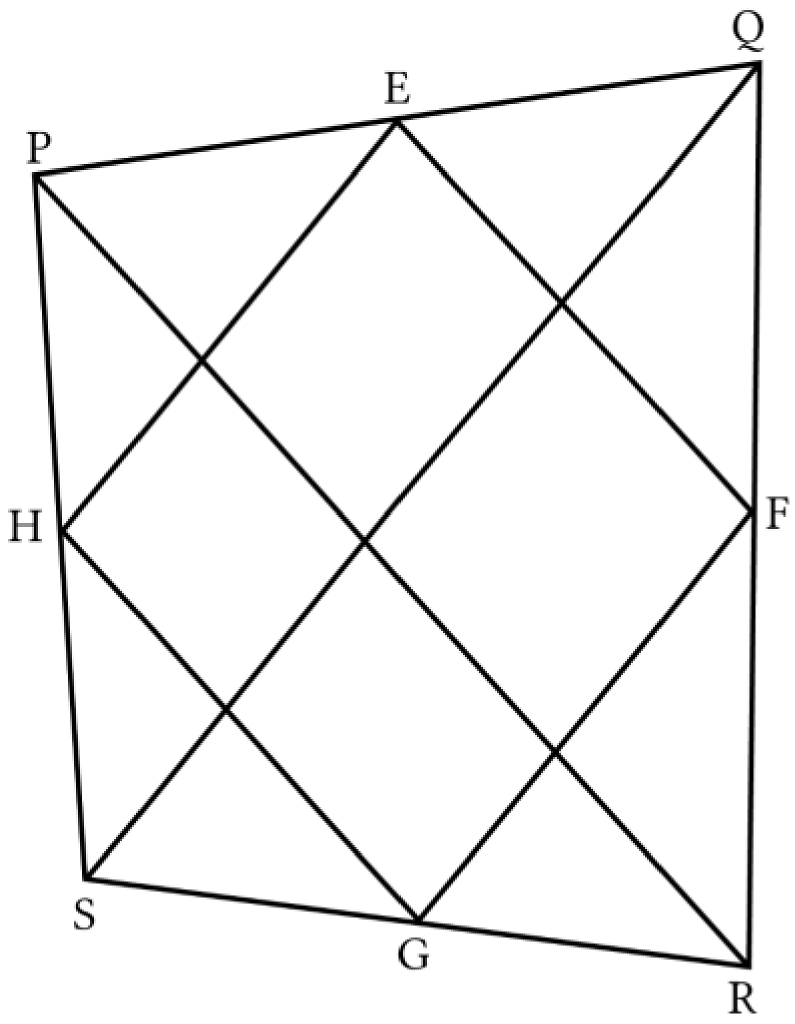
(b) PQRS is a parallelogram

(c) Diagonals of PQRS are perpendicular

(d) Diagonals of PQRS are equal

Solution:

d



Let EFGH be the rhombus formed by joining the midpoints of the sides of a quadrilateral PQRS.

So, $EF = FG = GH = HE$.

Now, in ΔPQR,

E and F are the midpoints of PQ and QR respectively.

So, $EF = \frac{1}{2}PR$ (By the midpoint theorem) .... (i)

Similarly, in ΔSQR,

F and G are the midpoints of RQ and RS respectively.

So, $GF = \frac{1}{2}SQ$ (By the midpoint theorem) .... (ii)

As, $EF = GF$ (Since EFGH is a rhombus),

$\therefore \frac{1}{2}PR = \frac{1}{2}SQ$ (Using (i) & (ii))

$\Rightarrow PR = SQ$

Hence, the diagonals of PQRS are equal.

Question: 6

If angles A, B, C and D of the quadrilateral ABCD, taken in order, are in the ratio

3:7:6:4, then ABCD is a

(a) Rhombus

(b) Parallelogram

(c) Trapezium

(d) Kite

Solution:

c

Given:

Ratio of the angles of quadrilateral ABCD is

3: 7: 6: 4.

Let the angles of the quadrilateral ABCD be 3a, 7a, 6a and 4a respectively.

Since the sum of all angles of a quadrilateral is 360°,

Therefore,

$$3a + 7a + 6a + 4a = 360^\circ$$
$$\Rightarrow 20a = 360^\circ$$
$$\Rightarrow a = \frac{360^\circ}{20} = 18^\circ$$

$\therefore$ The angles of the quadrilateral are

$$\angle A = 3 \times 18^\circ = 54^\circ$$
$$\angle B = 7 \times 18^\circ = 126^\circ$$
$$\angle C = 6 \times 18^\circ = 108^\circ$$

and $\angle D = 4 \times 18^\circ = 72^\circ$

Now, let's Produce AB to point M.

Then, $\angle ABC + \angle CBM = 180^\circ$ (Linear pair axiom)

$\Rightarrow 126^\circ + \angle CBM = 180^\circ$

$\Rightarrow \angle CBM = 180^\circ - 126^\circ = 54^\circ = \angle A$

Hence, $AD \parallel BC$ (Since, the corresponding angles are equal)

Now, the sum of co-interior angles,

$\angle A + \angle B = 126^\circ + 54^\circ = 180^\circ$

and $\angle C + \angle D = 108^\circ + 72^\circ = 180^\circ$

Hence, ABCD is a trapezium.

Question: 7

If bisectors of ∠A and ∠B of a quadrilateral ABCD intersect each other at P, bisectors of ∠B and ∠C at Q, bisectors of ∠C and ∠D at R and bisectors of ∠D and ∠A at S, then PQRS is a:

(a) Rectangle

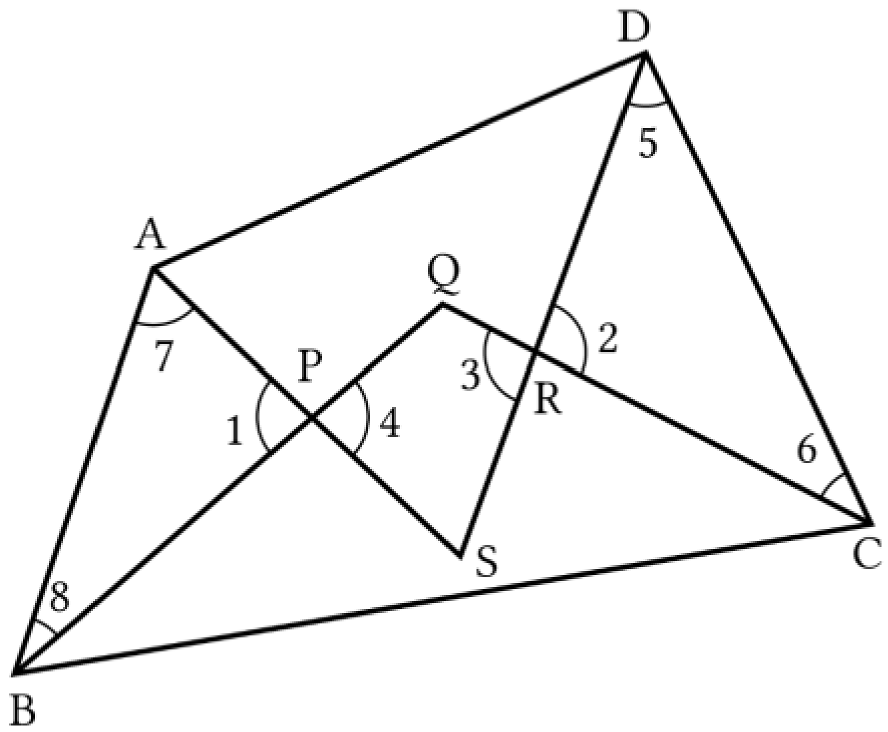
(b) Rhombus

(c) Parallelogram

(d) Quadrilateral whose opposite angles are supplementary

Solution:

d



Given: In quadrilateral ABCD, angle bisectors form a quadrilateral PQRS.

Since the sum of all angles in quadrilateral is 360°,

therefore, $\angle A + \angle B + \angle C + \angle D = 360°$.

On dividing both sides by 2,

$$\frac{1}{2}(\angle A+\angle B+\angle C+\angle D)=\frac{1}{2}\times 360^\circ$$

$$\Rightarrow \frac{1}{2}\angle A+\frac{1}{2}\angle B+\frac{1}{2}\angle C+\frac{1}{2}\angle D=180^\circ$$

$$\Rightarrow \angle 7+\angle 8+\angle 6+\angle 5=180^\circ \;\ldots\ldots\ldots\ldots(\text{i})$$

($\because$ AP, BP, CR and DR are angle bisectors of $\angle$A, $\angle$B, $\angle$C & $\angle$D respectively.)

Now, let's consider ΔCDR,

$\angle 6+\angle 5+\angle 2=180^\circ$ (Sum of all angles in a triangle)

$$\Rightarrow \angle 6+\angle 5=180^\circ-\angle 2 \;\ldots\ldots\ldots(\text{ii})$$

Similarly, on considering ΔABP,

$\angle 7+\angle 8+\angle 1=180^\circ$ (Sum of all angles in a triangle)

$$\Rightarrow \angle 7+\angle 8=180^\circ-\angle 1 \;\ldots\ldots\ldots(\text{iii})$$

Adding (ii) & (iii),

$$\Rightarrow \angle 7+\angle 8+\angle 6+\angle 5=180^\circ-\angle 1+180^\circ-\angle 2$$

$\Rightarrow 180^\circ=180^\circ-\angle 1+180^\circ-\angle 2$ (Using (i))

$$\Rightarrow \angle 1+\angle 2=180^\circ$$

But, $\angle 1=\angle 4$ and $\angle 2=\angle 3$ (Vertically opposite angles)

Hence, $\angle 3+\angle 4=180^\circ$.

Hence, PQRS is a quadrilateral whose opposite angles are supplementary.

Question: 8

If APB and CQD are two parallel lines, then the bisectors of the angles APQ, BPQ, CQP and PQD form

(a) A square

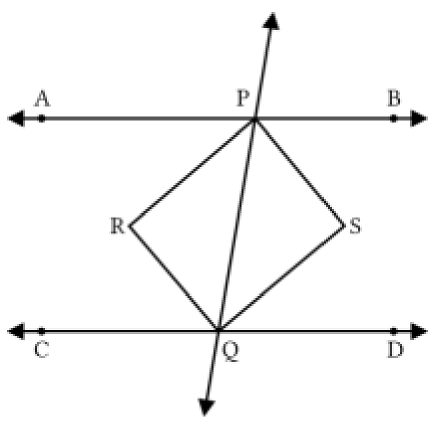
(b) A rhombus

(c) A rectangle

(d) Any other parallelogram

Solution:

c



Given: APB and CQD are two parallel lines.

Let the bisectors of angles APQ and CQP meet at a point R and the bisectors of angles DQP and BPQ meet at a point S.

Since APB is a line, $\angle APR + \angle RPQ + \angle QPS + \angle SPB = 180°$

(Linear pair axiom)

$\Rightarrow \angle RPQ + \angle RPQ + \angle QPS + \angle QPS = 180°$

(As PR and PS are angle bisectors for ∠APQ and ∠QPB respectively.)

$\Rightarrow 2\angle RPQ + 2\angle QPS = 180°$
$\Rightarrow 2(\angle RPQ + \angle QPS) = 180°$
$\Rightarrow (\angle RPQ + \angle QPS) = 90°$
$\Rightarrow \angle RPS = 90°$

Now,

$\angle APQ = \angle PQD$ ($\because$ $APB \parallel CQD$ and PQ is a traversal line)

$\Rightarrow \frac{1}{2}\angle APQ = \frac{1}{2}\angle PQD$
$\Rightarrow \angle RPQ = \angle PQS$

(PR and QS are angle bisectors for $\angle APQ$ and $\angle PQD$ respectively.)

$\Rightarrow PR \parallel QS$ (Alternate interior angle)

Similarly, $\angle BPQ = \angle PQC$ (Since $APB \parallel CQD$ and PQ is a traversal line)

$\Rightarrow PS \parallel RQ$

Hence □PRQS is a rectangle.

Question: 9

The figure obtained by joining the midpoints of the sides of a rhombus, taken in order, is:

(a) A rhombus

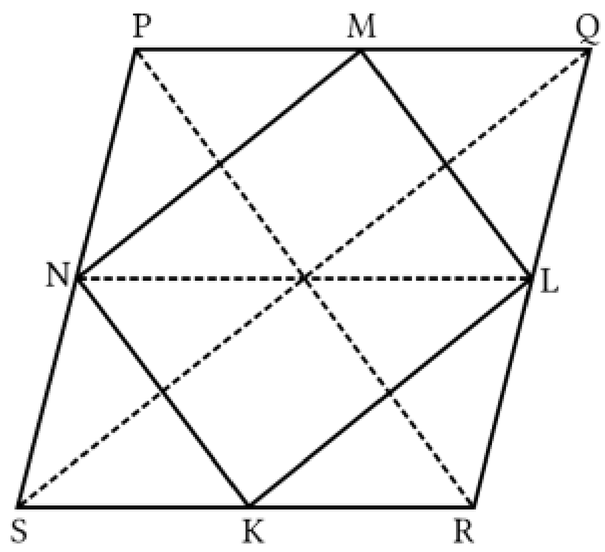
(b) A rectangle

(c) A square

(d) Any parallelogram

Solution:

b



Let KLMN be the quadrilateral which is formed by joining the mid points of the rhombus SRQP.

Let's join SQ, LN and KM.

Let us consider ΔPSQ.

Since N and M are mid points of sides PS and PQ respectively,

$\therefore NM \parallel SQ$

and $NM = \frac{1}{2} SQ$ ....(i)

(By the midpoint theorem)

Similarly, on considering ΔRSQ,

Since K and L are the midpoints of the sides SR and RQ respectively,

$\therefore\ KL \parallel SQ$

And $KL = \frac{1}{2}SQ$ ....(ii) (By the midpoint theorem)

Using (i) and (ii), we get

$KL \parallel NM$ and $KL = NM$ ....(iii)

Hence, KLMN is a parallelogram.

Also, NLRS is a parallelogram.

$\Rightarrow NL = SR$ ....(iv) (Opposite sides of a parallelogram)

Similarly, MKRQ is a parallelogram.

$\Rightarrow MK = QR$ (Opposite sides of a parallelogram)

$\Rightarrow MK = SR$ ($\because SR = QR$, sides of a rhombus)

⇒ MK=NL (Using (iv))

So, diagonals of a parallelogram are equal.

Hence, KLMN is a rectangle.

Question: 10

D and E are the mid-points of the sides AB and AC of ΔABC and O is any point on side BC. O is joined to A.

If P and Q are the midpoints of OB and OC respectively, then DEQP is:

(a) A square

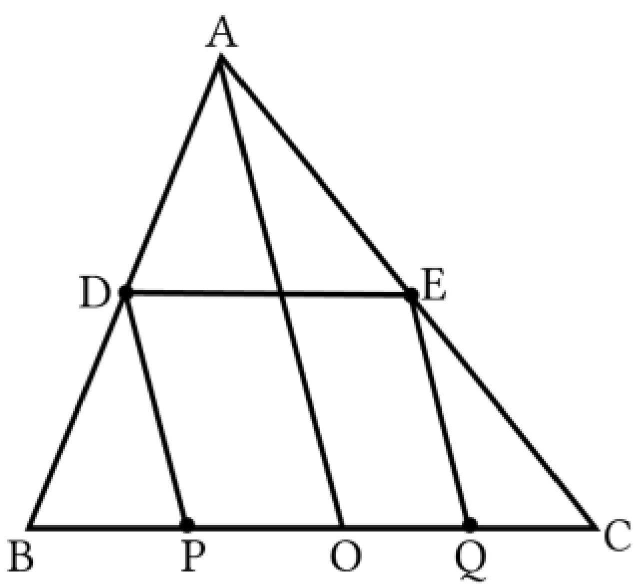
(b) A trapezium

(c) A rhombus

(d) A parallelogram

Solution:

d



We are given that D and E are the mid-points of the sides AB and AC of the ΔABC.

P and Q are the midpoints of OB and OC respectively.

Let's Join DE, DP and EQ.

In ΔABC,

$DE \parallel BC \quad \ldots\ldots\ldots(i)$

and $DE = \frac{1}{2}BC$

(∵ D & E are midpoint of AB and AC respectively, by the midpoint theorem.)

Then, $DE = \frac{1}{2}(BP + PO + OQ + QC) = \frac{1}{2}(2PO + 2OQ)$

$[\because BP = PQ \text{ and } OQ = QC]$

$\Rightarrow \ DE = PO + OQ$

$\Rightarrow \ DE = PQ \ \ldots\ldots\ldots\ldots \ (ii)$

Now, in ΔABO,

$DP \parallel AO \text{ and } DP = \frac{1}{2}AO \quad \ldots\ldots \ (iii)$

(∵ D and P are the midpoint of AB and BO respectively.)

Similarly, in ΔACO,

$EQ \parallel AO$ and $EQ = \frac{1}{2} AO$ ...... (iv)

($\because$ E and Q are midpoint of AC and OC respectively.)

Using eqn. (iii) & (iv), we get

$DP \parallel EQ$

and $DP = EQ$.

Using eqn. (i) & (ii), we get,

$DE \parallel BC \Rightarrow DE \parallel PQ$

and $DE = PQ$.

Hence, DEQP is a parallelogram.

Question: 11

The figure formed by joining the mid-points of the sides of a quadrilateral ABCD, taken in order, is a square only if,

(a) ABCD is a rhombus

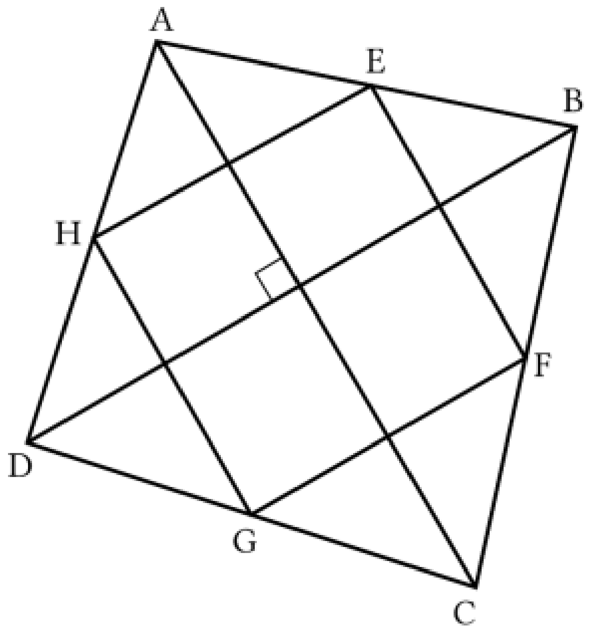
(b) Diagonals of ABCD are equal

(c) Diagonals of ABCD are equal and perpendicular

(d) Diagonals of ABCD are perpendicular.

Solution:

c



Let us consider EFGH be the quadrilateral formed by joining the midpoints of the sides of a quadrilateral ABCD. Then EFGH is a square.

$\therefore EF = FG = GH = HE$

and $HF = EG$

But, $EG = AD$ and $HF = CD$

$\therefore AD = CD$

Thus, all the sides of quadrilateral ABCD are equal.

Now, in ΔABC,

since E and F are midpoints of AB and BC respectively,

$EF = \frac{1}{2}AC$.......(i) (By midpoint theorem)

Similarly, in ΔADB,

since E and H are midpoints of AB and AD respectively,

$EH = \frac{1}{2}DB$ (By midpoint theorem)

$\Rightarrow EF = \frac{1}{2}DB \;....(ii)\; (\because EH = EF)$

From Eq. (i) and (ii),

$AC = DB$

Thus, all the sides of quadrilateral ABCD are equal and diagonals of ABCD are equal. Therefore, the quadrilateral ABCD is a square.

So, the diagonals of the quadrilateral are also perpendicular.

Question: 12

The diagonals AC and BD of a parallelogram ABCD intersect each other at the point O.

If $\angle DAC = 32^o$ and $\angle AOB = 70^o$, then $\angle$DBC is equal to:

(a) 24°

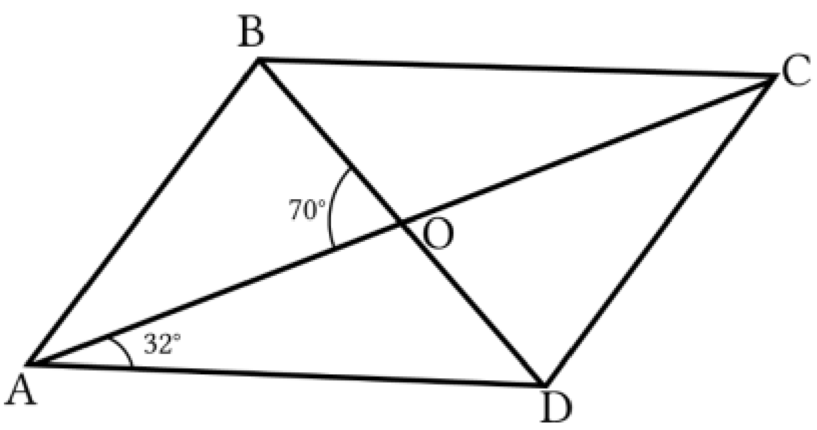
(b) 86°

(c) 38°

(d) 32°

Solution:

c



We are given that ABCD is a parallelogram, $\angle DAC = 32°$ and $\angle AOB = 70°$.

Since $\angle DAC = \angle ACB = 32^\circ$ (Alternate interior angle),

also, $\angle ACB = \angle OCB$

$\Rightarrow \angle OCB = 32^\circ$

Now, let's consider ΔBOC.

$\angle$AOB is an exterior angle to ΔBOC.

$\Rightarrow \angle OBC + \angle OCB = \angle AOB$ (Exterior angle property)

$\Rightarrow \angle OBC = \angle AOB - \angle OCB = 70^\circ - 32^\circ = 38^\circ$

Also, $\angle DBC = \angle OBC$

Thus, $\angle DBC = 38^\circ$

Question: 13

Which of the following is not true for a parallelogram?

(a) Opposite sides are equal

(b) Opposite angles are equal

(c) Opposite angles are bisected by the diagonals

(d) Diagonals bisect each other

Solution:

c

In a parallelogram, the opposite sides are equal, the opposite angles are equal, the diagonals bisect each other but the opposite angles are not bisected by the diagonals.

Question: 14

D and E are the midpoints of the sides AB and AC respectively of triangle ABC. DE is produced to F. To prove that CF is equal and parallel to DA, we need additional information which is:

(a) $\angle DAE = \angle EFC$

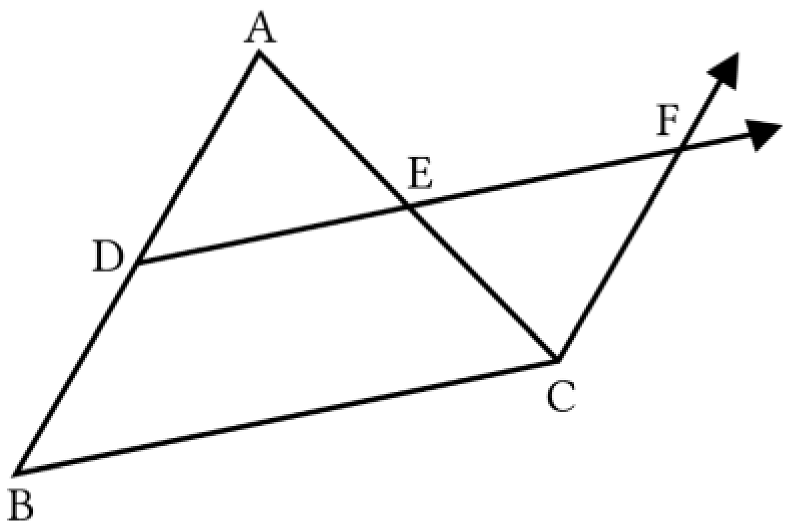
(b) $AE = EF$

(c) $DE = EF$

(d) $\angle ADE = \angle ECF$

Solution:

c



In ΔABC, since, D and E are the midpoints of AB and AC respectively, therefore, $DE \parallel BC$ and $DE = \frac{1}{2}BC$.

$\Rightarrow DF \parallel BC$

and $2DE = BC$ ....(i)

Suppose, $DE = EF$ then $DF = 2DE$.

Then, using Eq. (i) we get, $DF \parallel BC$

and $DF = BC$.

Hence, the quadrilateral DBCF is a parallelogram.

$\Rightarrow DB = CF$ and $DB \parallel CF$ (Property of a parallelogram)

$\Rightarrow AD = CF$ and $AB \parallel CF$ ($\because$ D is the midpoint of AB)

$\Rightarrow AD = CF$ and $AD \parallel CF$ ($\because$ AD is a part of line AB).

Hence, option(c) $DE = EF$ is the additional information that we need to prove the desired result.

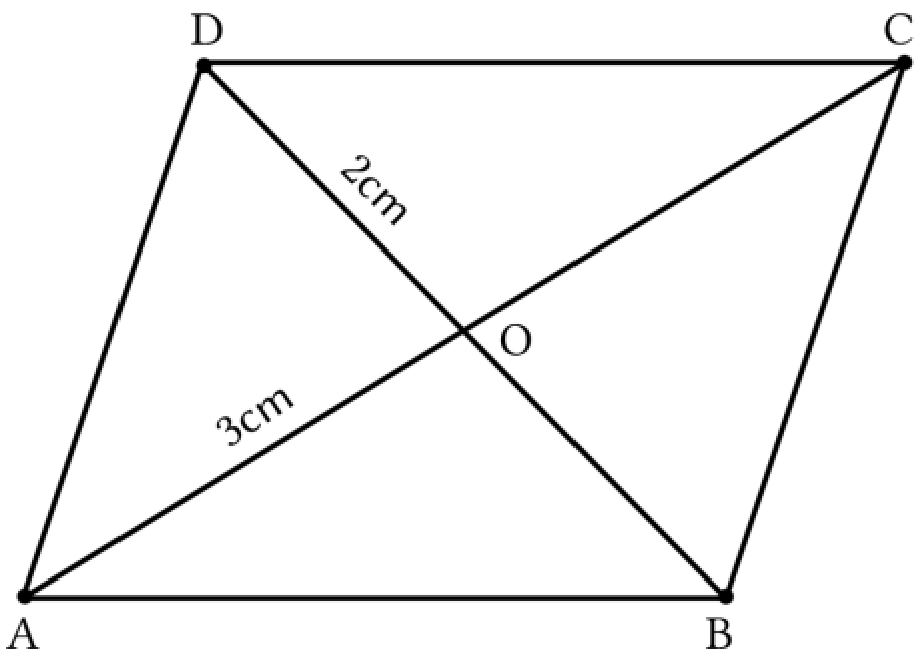
Exercise 8.2

Question: 1

Diagonals AC and BD of a parallelogram ABCD intersect each other at O.

If $OA = 3cm$ and $OD = 2cm$, determine the lengths of AC and BD.

Solution:



We are given that $\square ABCD$ is a parallelogram,

$OA = 3cm$ and $OD = 2cm$.

We need to find the length of AC and BD.

Since $\square ABCD$ is a parallelogram and the diagonals of a parallelogram bisects each other,

$\therefore AC = 2OA = 2 \times 3\text{cm} = 6\text{cm}$

and, $BD = 2OD = 2 \times 2\text{cm} = 4\text{cm}.$

Hence it is proved.

Question: 2

Diagonals of a parallelogram are perpendicular to each other. Is this statement true? Give reason for your answer.

Solution:

No; if the parallelogram is a rhombus then the diagonals are perpendicular to each other. Otherwise, in all parallelograms, diagonals bisect each other.

Question: 3

Can the angles 110°, 80°, 70° and 95° be the angles of a quadrilateral? Why or why not?

Solution:

We know that the sum of all angles of a quadrilateral is 360°.

Here,

$110° + 80° + 70° + 95° = 355° \neq 360°$

So, $110°, 80°, 70°, 95°$ cannot be the angles of a quadrilateral.

Question: 4

In quadrilateral ABCD, $\angle A + \angle D = 180°$. What special name can be given to this quadrilateral?

Solution:

Since

$\angle A + \angle D = 180°$,

$\Rightarrow AB \parallel DC$ (Sum of co-interior angles is $180°$)

$\therefore \square ABCD$ can be called a trapezium.

Question: 5

All the angles of a quadrilateral are equal. What special name is given to this quadrilateral?

Solution:

Given, all the angles of a quadrilateral are equal.

Let each angle of the quadrilateral be ‘x’.

Then, $x + x + x + x = 360^\circ$ (Sum of all angles of a quadrilateral)

$$\Rightarrow 4x = 360^\circ$$
$$\Rightarrow x = 90^\circ$$

Since all the angles of the quadrilateral are 90º, the given quadrilateral is a rectangle.

Question: 6

Diagonals of a rectangle are equal and perpendicular to each other. Is this statement true? Give reason for your answer.

Solution:

No; Diagonals of a rectangle are not perpendicular to each other.

Question: 7

Can all the four angles of a quadrilateral be obtuse angles? Give reason for your answer.

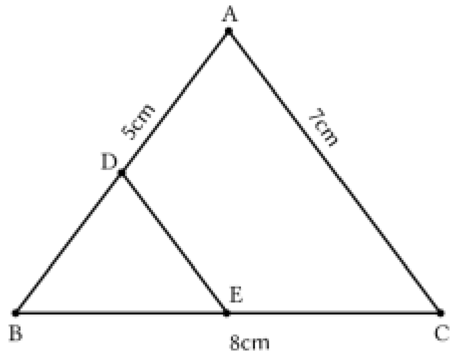
Solution:

No; because sum of the angles of a quadrilateral is 360°. A quadrilateral can have a maximum of three obtuse angles.

Question: 8

In $\Delta ABC,\ AB = 5cm,\ BC = 8cm$ and $CA = 7cm$. If D and E are the midpoints of AB and BC, determine the length of DE.

Solution:



Given:

In $\Delta ABC,\ AB = 5cm,\ BC = 8cm$ and $CA = 7cm$.

D and E are respectively the midpoints of AB and BC.

Let's join DE.

By the midpoint theorem,

DE∥AC and DE = $\frac{1}{2}$AC

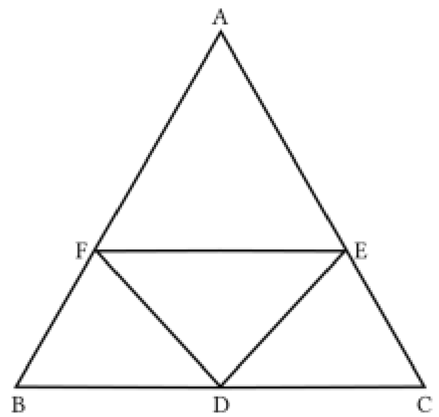
$= \frac{1}{2} \times 7$

$= 3.5\,cm$

Question: 9

In the following figure, it is given that BDEF and FDCE are parallelograms.

Can you say that $BD = CD$? Why or why not?



Solution:

It is given that ▱BDEF is a || *gram*,

$\therefore BD = FE.....(i)$ (Opposite sides of || gram)

Also, ▱FDCE is a || *gram*,

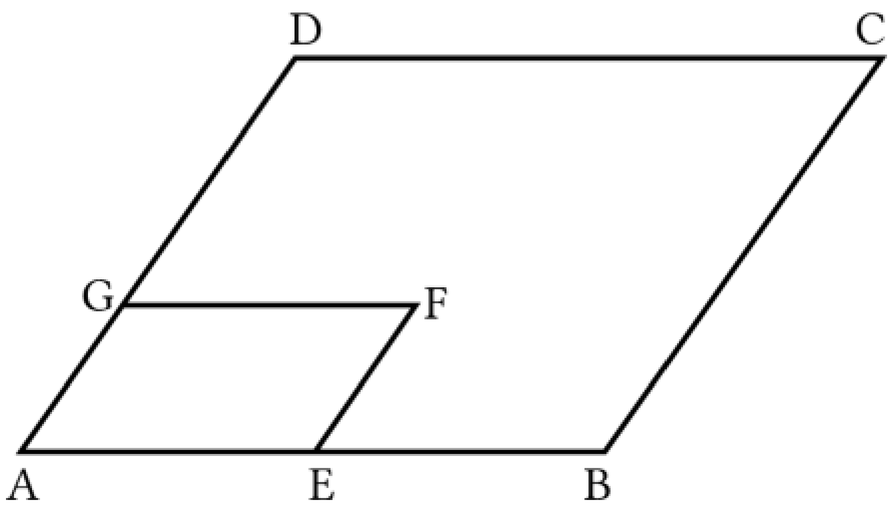
$\therefore$ DC = FE ...(ii)

From eq. (i) and (ii)

BD = CD

Question: 10

In the given figure, ABCD and AEFG are two parallelograms. If ∠C = 55°, determine ∠F.



Solution:

Since ABCD is a || gram,

$\therefore \angle A = \angle C = 55^\circ$ .....(i) (Opposite angles of a parallelogram are equal)

Again, AEFG is a || gram,

$\therefore \angle F = \angle A = 55°$ (Using Eq. (i)).

Question: 11

Can all the angles of a quadrilateral be acute angles? Give reason for your answer.

Solution:

No; the angle sum of a quadrilateral is 360°. Therefore, a quadrilateral should have at least one obtuse angle.

Question: 12

Can all the angles of a quadrilateral be right angles? Give reason for your answer.

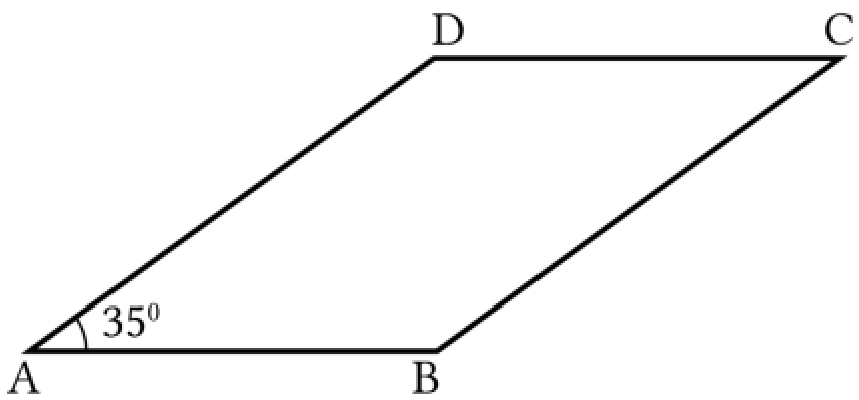
Solution:

Yes, because the angle sum will be 360°, which is a required property of a quadrilateral.

Question: 13

Diagonals of a quadrilateral ABCD bisect each other. If $\angle A = 35°$, determine $\angle B$.

Solution:



Since diagonals of the given quadrilateral bisect each other,

∴ ABCD is a || gram

Now,

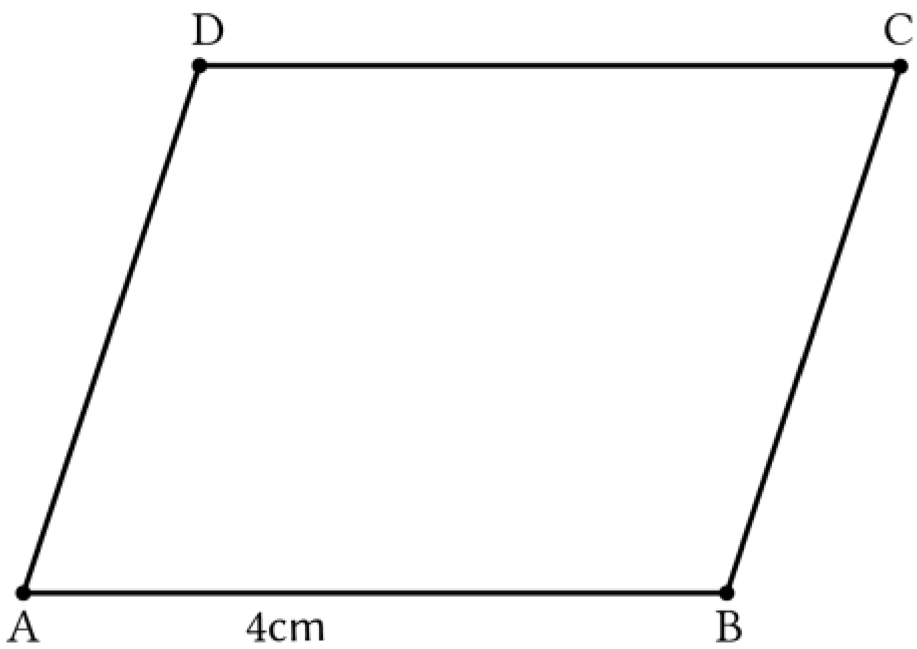
$\angle DAB$ and $\angle ABC$ are consecutive interior angles as $AD \parallel BC$ and AB is a transversal and we know that some of the consecutive interior angles $180°$.

$$\therefore \angle DAB + \angle ABC = 180°$$
$$\Rightarrow \angle ABC = 180° - \angle DAB = 180° - 35°$$
$$\Rightarrow \angle ABC = 145°$$

Question: 14

Opposite angles of a quadrilateral ABCD are equal. If $AB = 4cm$, determine CD.

Solution:



Given $\angle A = \angle C,\ \angle B = \angle D$

$\therefore$ ▱ABCD is a ||gram.

$\therefore$ $CD = AB = 4cm$ (Opposite sides of a parallelogram)

Exercise 8.3

Question: 1

One angle of a quadrilateral is of 108° and the remaining three angles are equal.

Find each of the three equal angles.

Solution:

Let ABCD be a quadrilateral such that

$\angle A = 108^\circ$ and $\angle B = \angle C = \angle D$.

Let $\angle B = \angle C = \angle D = x$

Now by the angle sum property we have,

$$\angle A + \angle B + \angle C + \angle D = 360^\circ$$
$$\Rightarrow \angle A + x + x + x = 360^\circ$$
$$\Rightarrow 108^\circ + 3x = 360^\circ$$
$$\Rightarrow 3x = 360^\circ - 108^\circ$$
$$\Rightarrow 3x = 252^\circ$$
$$\Rightarrow x = \frac{252^\circ}{3}$$
$$x = 84^\circ$$

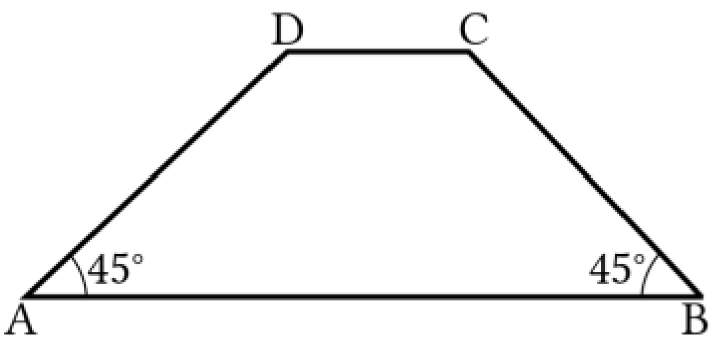
Hence, the measure of each of the equal angle is 84°.

Question: 2

ABCD is a trapezium in which $AB \parallel DC$ and $\angle A = \angle B = 45^o$.

Find angles C and D of the trapezium.

Solution:



We are given that ABCD is a trapezium and

$\angle A = \angle B = 45^o$

And

$DC \parallel AB$

So, $\angle A$ and $\angle D$ are co-interior angles.

$\Rightarrow \angle A + \angle D = 180°$ (Sum of co-interior angles)

$\Rightarrow 45° + \angle D = 180°$

$\Rightarrow \angle D = 180° - 45° = 135°$

Similarly, $\angle B$ and $\angle C$ are co-interior angles.

$\Rightarrow \angle B + \angle C = 180^\circ$ (Sum of co-interior angles)

$\Rightarrow 45^\circ + \angle C = 180^\circ$

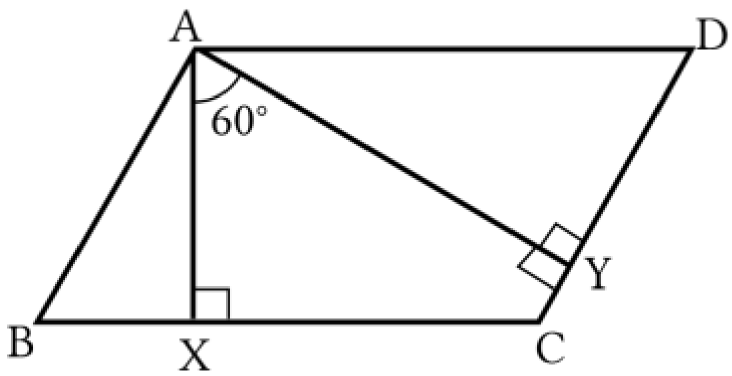
$\Rightarrow \angle C = 180^\circ - 45^\circ = 135^\circ$

Hence, the angles C and D are 135° each.

Question: 3

The angle between two altitudes of a parallelogram through the vertex of an obtuse angle of the parallelogram is 60°. Find the angles of the parallelogram.

Solution:



Let $ABCD$ is a parallelogram and $AX \perp BC$, $AY \perp CD$

and $\angle XAY = 60^\circ$.

In $\square AXCY$,

using the angle sum property of a quadrilateral,

$\angle XAY + \angle AYC + \angle C + \angle CXA = 360°$

$\Rightarrow\ 60° + 90° + \angle C + 90° = 360°$

$\Rightarrow\ 240° + \angle C = 360°$

$\Rightarrow\ \angle C = 360° - 240°$

$\therefore\ \angle C = 120°$

$\therefore\ \angle A = \angle C = 120°$

(Opposite angle of parallelogram)

Now,

$\therefore\ \angle C + \angle D = 180°$ (Adjacent angles of a parallelogram)

$\Rightarrow 120° + \angle D = 180°$

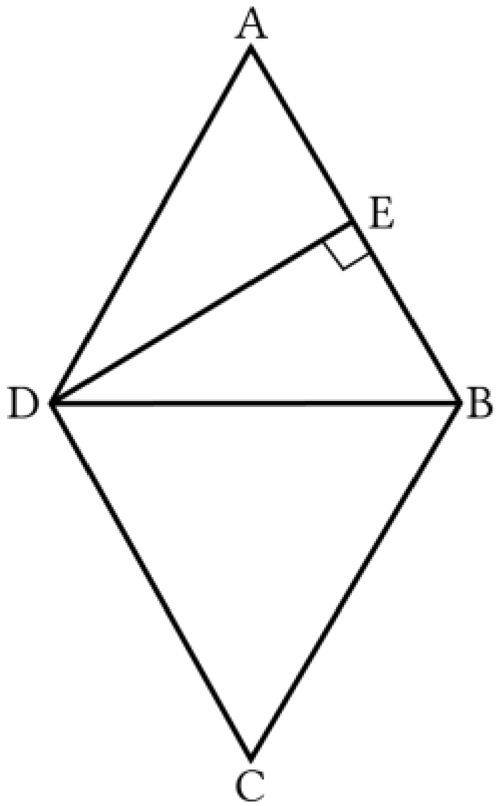
$\Rightarrow \angle D = 180° - 120° = 60°$

$\therefore \angle B = \angle D = 60°$ (Opposite angles of parallelogram are equal).

Question: 4

ABCD is a rhombus in which altitude from D to side AB bisects AB. Find the angles of the rhombus.

Solution:



Given: ABCD is a rhombus and DE is the altitude on AB

such that $AE = EB$.

In ΔAED and ΔBED,

$DE = DE$ (Common side)

$\angle DEA = \angle DEB (90^\circ)$

$AE = EB$ (Given)

$\therefore \Delta AED \cong \Delta BED$ (By SAS congruence)

$\Rightarrow AD = BD\ (C.P.C.T)$

Also, $AD = AB$

Then, $AD = AB = BD$

Thus, ΔABD is an equilateral triangle.

$\therefore \angle A = 60^\circ$
$\Rightarrow \angle C = \angle A = 60^\circ$

[Opposite angles of rhombus are equal]

Now, $\angle ADC + \angle DAB = 180^\circ$

[Sum of the adjacent angles of a rhombus is supplementary]

$\therefore \angle ADC + 60^\circ = 180^\circ$
$\Rightarrow \angle ADC = 180^\circ - 60^\circ$
$\therefore \angle ADC = 120^\circ$
$\therefore \angle ABC = \angle ADC = 120^\circ$

[Opposite angles of a rhombus are equal]

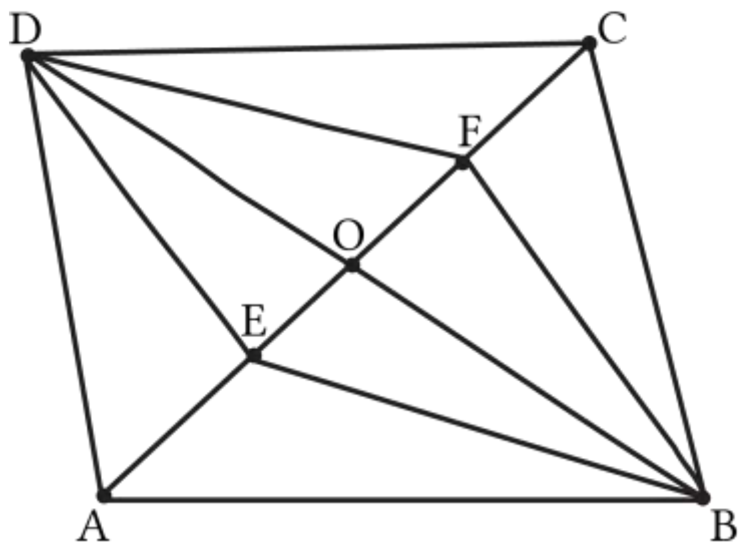
Thus, the angles of rhombus are 60°, 120°, 60° and 120°.

Question: 5

E and F are points on diagonal AC of a parallelogram ABCD such that $AE = CF$**.**

Show that BFDE is a parallelogram.

Solution:



Since ABCD is a$\parallel$ gram, the diagonals of a parallelogram bisect each other.

$\therefore OD = OB \ldots.(i)$

$OA = OC \ldots.(ii)$

$\mathrm{AE} = \mathrm{CF} \ldots.(\mathrm{iii})$ (Given)

$(\mathrm{iii}) - (\mathrm{ii})$

$\Rightarrow \mathrm{OA} - \mathrm{AE} = \mathrm{OC} - \mathrm{CF}$

$\Rightarrow \mathrm{OE} = \mathrm{OF} \ldots.(\mathrm{iv})$

Hence, in $\square \mathrm{BFDE}$ diagonals bisect each other.

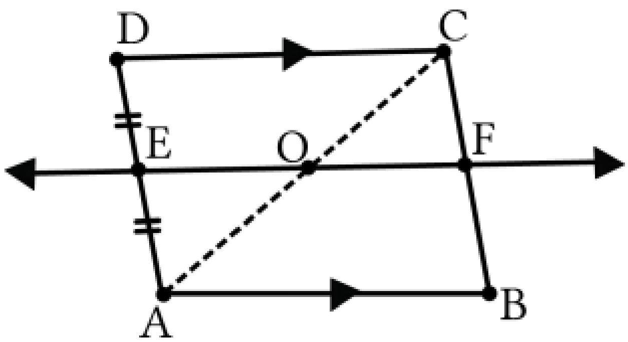
Therefore, $BFDE$ is parallelogram.

Question: 6

E is the midpoint of the side AD of the trapezium ABCD with $AB \parallel DC$. A line through E drawn parallel to AB intersects BC at F. Show that F is the midpoint of BC.

[Hint: Join AC]

Solution:



Given: ABCD is a trapezium with $AB \parallel DC$.

E is midpoint of AD and $EF \parallel AB$.

Since AB || DC and EF || AB,

$\Rightarrow EF \parallel DC$

Now, in ΔADC, E is midpoint of AD and

$EF \parallel DC$

$\Rightarrow EO \parallel DC$

$\Rightarrow$ O is midpoint of AC (Converse of the midpoint theorem)

In ΔCAB, O is the midpoint of AC (As proved above)

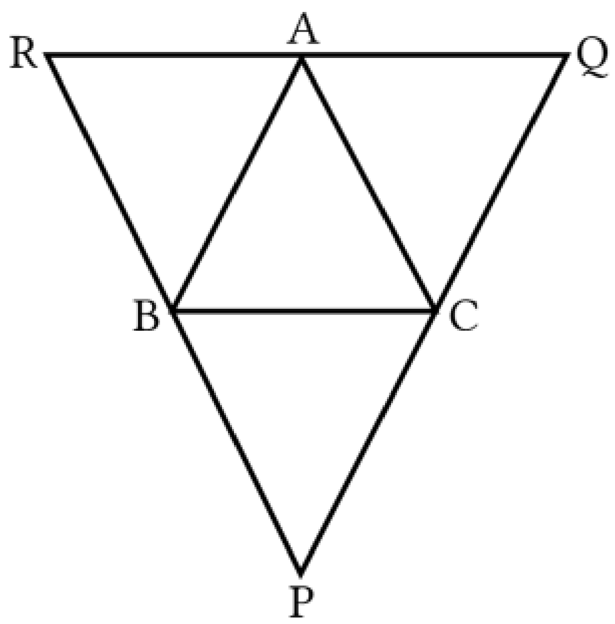
And $OF \parallel AB$ ($\because EF \parallel AB$ and OF lies on EF.)

$\Rightarrow$ F is the midpoint of BC. (Converse of the midpoint theorem)

Hence, we proved that $CF = FB$.

Question: 7

Through A, B and C, lines RQ, PR and QP have been drawn, respectively parallel to sides BC, CA and AB of a ΔABC as shown in the figure. Show that $BC = \frac{1}{2}QR$.



Solution:

Given: $AB \parallel QP$, $BC \parallel RQ$ and $CA \parallel PR$.

We need to show that $BC = \frac{1}{2}QR$.

Let us consider $\square RBCA$.

Since $RA \parallel BC$ and $BR \parallel CA$ (Given),

$\therefore$ $\square RBCA$ is a parallelogram,

so, $RA = BC....(i)$ [Opposite sides of parallelogram]

Similarly, on considering $\square BCQA$,

Since, $AQ \parallel BC$ and $AB \parallel CQ$

$\therefore$ $\square BCQA$ is parallelogram

so, $AQ = BC....(ii)$ [Opposite side of a parallelogram]

Adding (i) and (ii), we get

$\Rightarrow QR = 2BC$

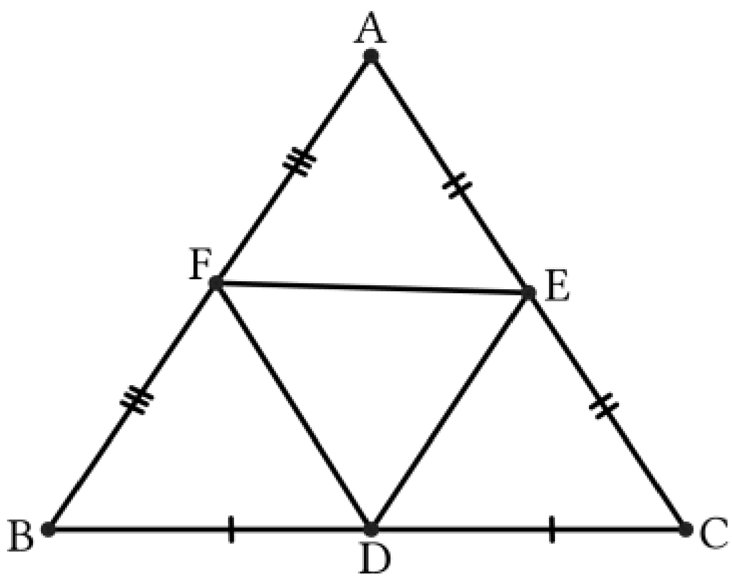
$\Rightarrow BC = \frac{1}{2}QR$

Hence, it is proved.

Question: 8

D, E and F are the midpoints of the sides BC, CA and AB, respectively of an equilateral triangle ABC. Show that □DEF is also an equilateral triangle.

Solution:



Since FE is a line segment joining the midpoints of sides AB and AC respectively,

$\therefore\ FE = \frac{1}{2}BC\ ....(i)$ (By midpoint theorem)

Similarly, $DE = \frac{1}{2}AB\ ...(ii)$

($\because$ D & E are midpoint of BC & CA respectively.)

and $DF = \frac{1}{2} AC....(iii)$

($\because$ D & F are midpoint of BC & AB respectively.)

But, $AB = BC = CA$ (Sides of an equilateral Δ)

$\Rightarrow \frac{1}{2} AB = \frac{1}{2} BC = \frac{1}{2} CA$ .....(iv) (Dividing by 2)

Using (i), (ii), (iii) in (iv)

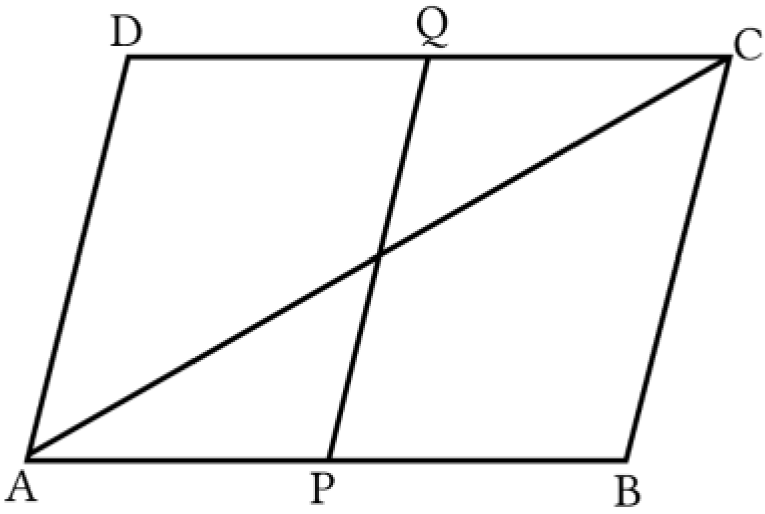
$DE = EF = FD$

$\therefore \Delta DEF$ is an equilateral triangle.

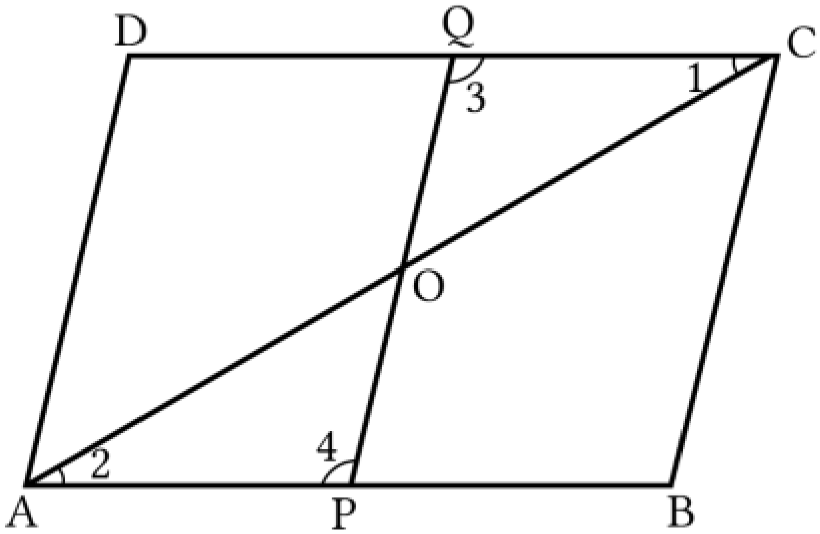
(A)

Question: 9

Points P and Q have been taken on opposite sides AB and CD, respectively of a parallelogram ABCD such that $AP = CQ$ as shown below. Show that AC and PQ bisect each other.



Solution:



Let us consider ΔAOP and ΔQOC

$\angle 1 = \angle 2$ ($\because$ $DC \parallel AB$ and alternate interior angles are equal)

$AP = CQ$ (Given)

$\angle 3 = \angle 4$ ($\because$ $AB \parallel CD$ and QP is a transversal)

$\Delta APO \cong \Delta CQO$ by ASA congruency rule

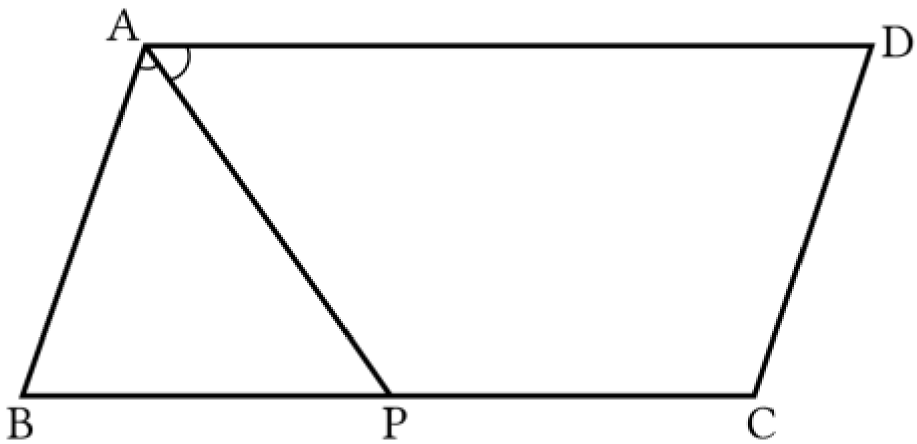
$\Rightarrow OA = OC \left(C.P.C.T\right)$

and $OP = OQ$.

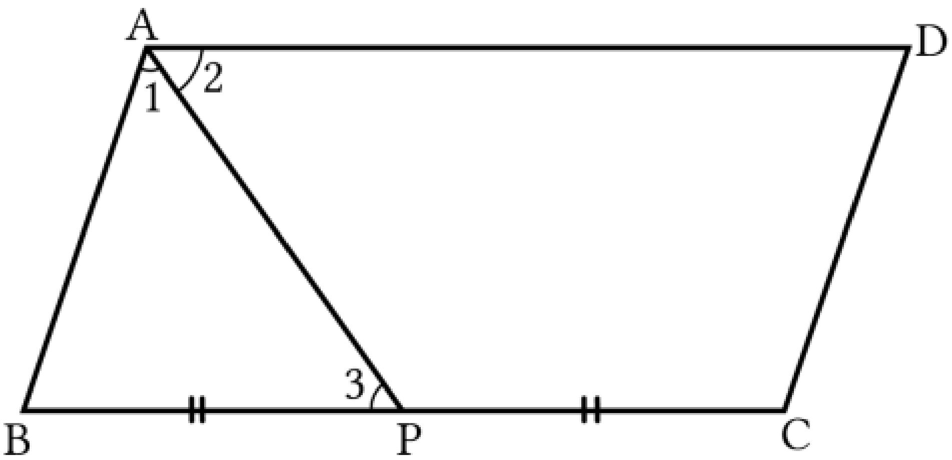
Hence we have proved that AC and PQ bisect each other.

Question 10

In the given below figure, P is the midpoint of side BC of a parallelogram ABCD such that $\angle BAP = \angle DAP$. **Prove that** $AD = 2CD$.



Solution:



Since $AD \parallel BC$ (Opposite sides of parallelogram $ABCD$)

and AP is transversal,

$\Rightarrow \angle 2 = \angle 3$ (Alternate interior angles)

But, $\angle 1 = \angle 2$ (Given)

$\therefore \angle 1 = \angle 3$

In ΔABP, $\angle 1 = \angle 3$

$\Rightarrow BP = AB$ (Sides opposite to equal angles are equal)

$\Rightarrow \frac{1}{2}BC = AB$ ($\because P$ is the midpoint of side BC)

$\Rightarrow \frac{1}{2}AD = AB$ ($\because BC = AD$, opposite sides of a parallelogram)

$\Rightarrow \frac{1}{2}AD = CD$ ($\because AB = CD$, opposite sides of a parallelogram)

$\Rightarrow AD = 2CD$

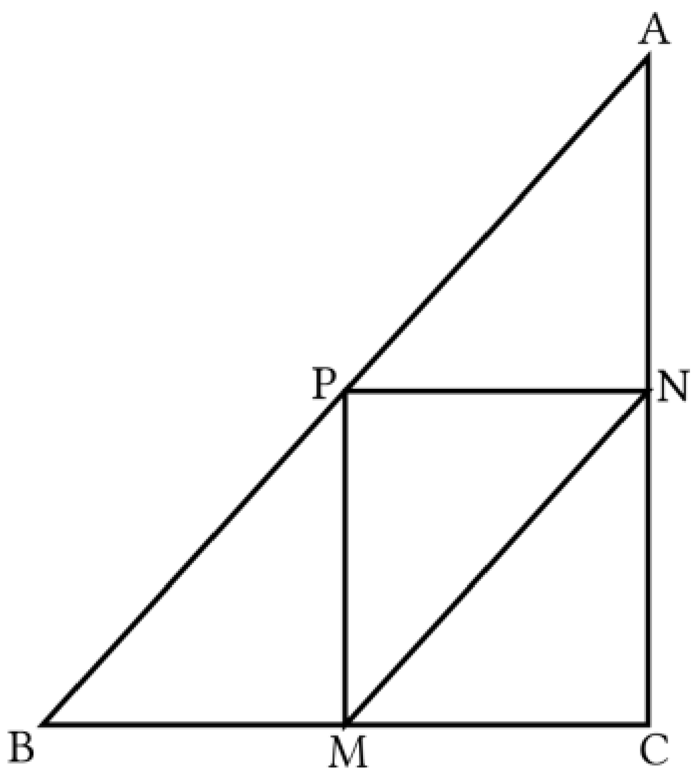
Hence, it is proved.

Exercise 8.4

Question: 1

A square is inscribed in an isosceles right triangle so that the square and the triangle have one angle common. Show that the vertex of the square opposite the vertex of the common angle bisects the hypotenuse.

Solution:



Given: An isosceles right ΔABC such that $\angle ACB = 90°$ and $BC = AC$.

A square CMPN is inscribed in it.

We need to prove that P bisects the hypotenuse AB that is $AP = PB$.

Since CMPN is a square,

$\therefore CM = MP = PN = CN$ (All sides are equal)

Also, ΔABC is isosceles with $AC = BC$

$\Rightarrow AN + NC = CM + MB$
$\Rightarrow AN = MB \quad (\because CN = CM)$

Let's consider ΔANP and ΔBMP.

AN = MB (As proved above)

$\angle ANP = \angle PMB = 90°$

$PN = PM$ ($\because$ CMPN is a square)

$\therefore \Delta ANP \cong \Delta BMP$ (By SAS congruency)

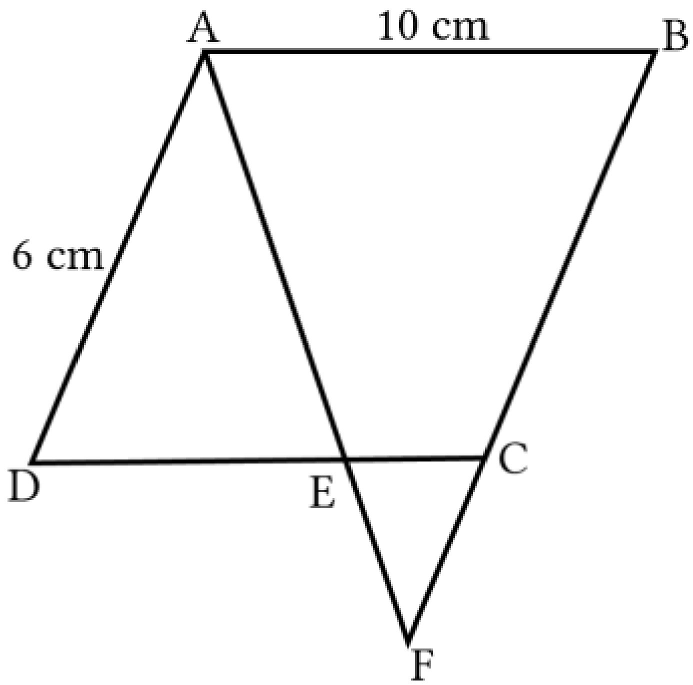
$\Rightarrow AP = PB$ (By CPCT).

Hence, it is proved.

Question: 2

In a parallelogram $ABCD$, $AB = 10cm$ and $AD = 6cm$. The bisector of $\angle A$ meets DC at E. AE and BC produced to meet at F. Find the length of CF.

Solution:



Given: ABCD is a parallelogram and $AB = 10$ cm; $AD = 6$ cm.

AE is the angle bisector of angle A.

Now,

$\angle DAF = \angle AFB$ (Alternate interior angle)

But, $\angle DAF = \angle FAB$ (Since AE is the angle bisector of $\angle A$)

$\Rightarrow \angle AFB = \angle FAB$

$\therefore$ In ΔABF,

$\Rightarrow BF = AB = 10cm$ ($\because$ Sides opposite to equal angle are equal)

But, $BC = AD = 6cm$ (Opposite sides of a parallelogram are equal)

Now, $CF = BF - BC$

$= 10 - 6$

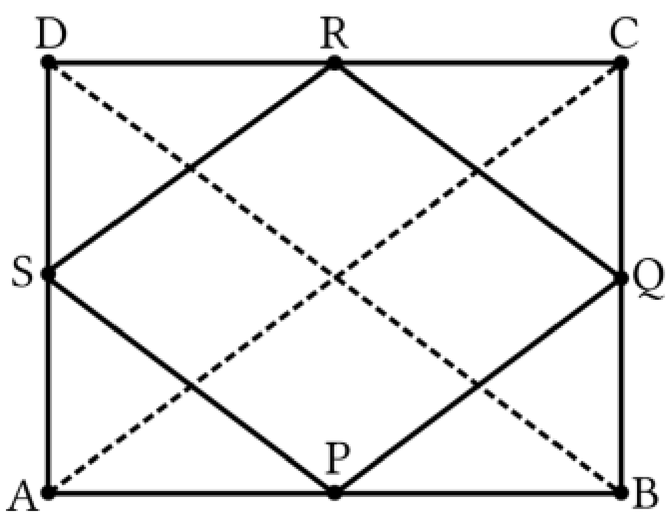
$= 4cm$

Hence, the length of CF=4 cm.

Question: 3

P, Q, R and S are respectively the midpoints of the sides AB, BC, CD and DA of a quadrilateral ABCD in which $AC = BD$. **Prove that PQRS is a rhombus.**

Solution:



Given: ABCD is a quadrilateral in which P, Q, R and S are the midpoints of AB, BC, CD and DA respectively. Let's join PQ, QR, RS and SP.

Also, $AC = BD$.

Now, in ΔABC,

P and Q are the midpoints of AB and BC respectively.

$\therefore PQ \parallel AC$ and $PQ = \frac{1}{2}AC$ .....(i) (By the midpoint theorem)

Similarly, in ΔADC,

S and R are the midpoints of AD and DC respectively.

$\therefore SR \parallel AC$ and $SR = \frac{1}{2}AC$ .....(ii) (By the midpoint theorem)

Similarly, in ΔDBC,

$\therefore RQ \parallel BD$ and $RQ = \frac{1}{2}BD$ .....(iii) (By the midpoint theorem)

And, in ΔDBA,

$\therefore SP \parallel BD$ and $SP = \frac{1}{2}BD$ .....(iv) (By the midpoint theorem)

Now, $AC = BD$ (Given)

Therefore, from eqn. (i), (ii), (iii) & (iv), we get

$PQ \parallel SR \parallel RQ \parallel SP$ and $PQ = SR = RQ = SP.$

Therefore, each side of the quadrilateral PQRS are equal and parallel.

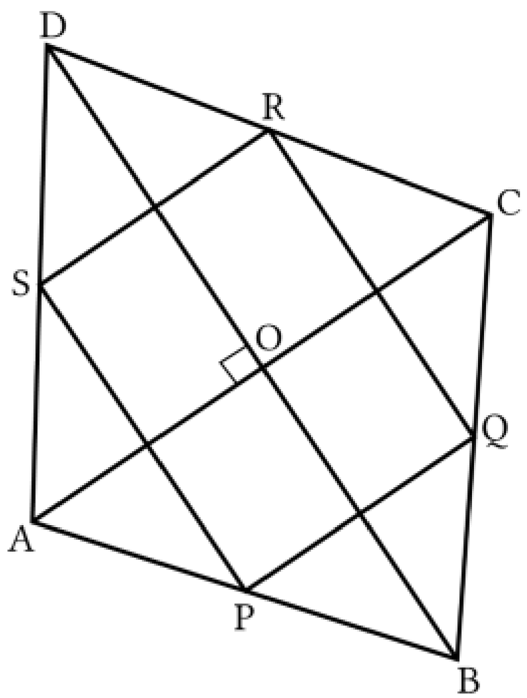
Hence,

PQRS is a rhombus.

Question: 4

P, Q, R and S are respectively the mid-points of the sides AB, BC, CD and DA of a quadrilateral ABCD such that AC $\perp$ BD. Prove that PQRS is a rectangle.

Solution



Given: P, Q, R and S are respectively the midpoints of the sides AB, BC, CD and DA of a quadrilateral ABCD such that $AC \perp BD$.

In ΔABC,

P and Q are the midpoints of AB and BC respectively.

$\therefore PQ \parallel AC$ and $PQ = \frac{1}{2}AC$ .....(i) (By the midpoint theorem)

Similarly, in ΔADC,

S and R are the midpoints of AD and DC respectively.

$\therefore SR \parallel AC$ and $SR = \frac{1}{2}AC$ .....(ii) (By the midpoint theorem)

From (i) and (ii), we get

$PQ \parallel SR$ and $PQ = SR.$

Similarly, in ΔDBC,

$RQ \parallel BD$ and $RQ = \frac{1}{2}BD$ .....(iii) (By the midpoint theorem)

In ΔDBA,

$\therefore SP \parallel BD$ and $SP = \frac{1}{2}BD$ .....(iv) (By the midpoint theorem)

From (iii) and (iv), we get,

$\therefore RQ \parallel SP$ and $RQ = SP$

Hence ▱PQRS is a parallelogram.

Now,

$AC \perp BD$ (Given)

And, $PQ \parallel AC$ (From eqn. (i))

$\Rightarrow PQ \perp BD$

But, $RQ \parallel BD$ (From eqn. (iii))

$\Rightarrow PQ \perp RQ$

Similarly, we can show that

$RQ \perp SR,$

$SR \perp PS$

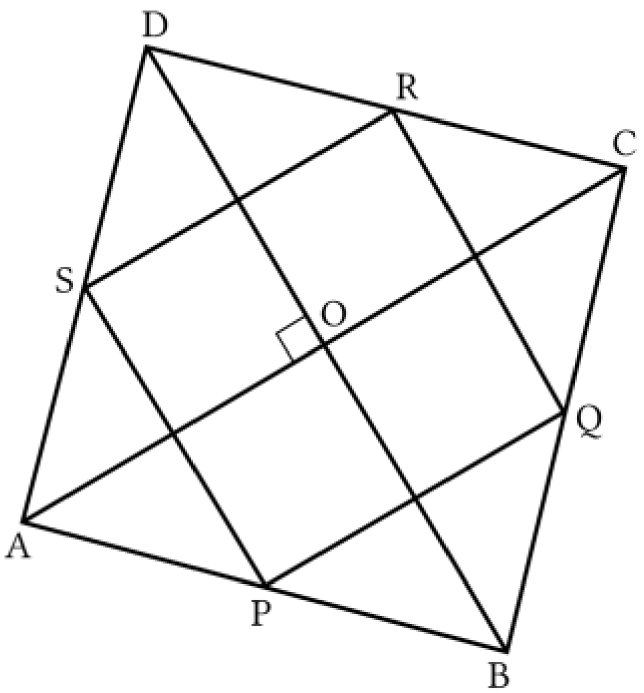
And $PS \perp PQ$

Hence, it is proved that $\square PQRS$ is a rectangle.

Question 5

P, Q, R and S are respectively the midpoints of sides AB, BC, CD and DA of quadrilateral ABCD in which $AC = BD$ and AC $\perp$BD. Prove that PQRS is a square.

Solution:



In ΔABC,

P and Q are the midpoints of AB and BC respectively.

$\therefore PQ \parallel AC$ and $PQ = \frac{1}{2}AC$ .....(i) (By the midpoint theorem)

Similarly, in ΔADC,

S and R are the midpoints of AD and DC respectively.

$\therefore SR \parallel AC$ and $SR = \frac{1}{2}AC$ .....(ii) (By the midpoint theorem)

From (i) and (ii), we get

$PQ \parallel SR$ and $PQ = SR$

Similarly, in ΔDBC,

$RQ \parallel BD$ and $RQ = \frac{1}{2}BD$ .....(iii) (By the midpoint theorem)

and, in ΔDBA,

$SP \parallel BD$ and $SP = \frac{1}{2}BD$ .....(iv) (By midpoint theorem)

From (iii) and (iv), we get,

$RQ \parallel SP$ and $RQ = SP$

Hence, $\square PQRS$ is a parallelogram.

Now, $AC = BD$ (Given)

$\therefore PQ = QR = RS = SP$ (Using (i), (ii), (iii) & (iv))

Now,

$AC \perp BD$ (Given)

And, PQ || AC (From eqn. (i))

$\Rightarrow PQ \perp BD$

But, $RQ \parallel BD$ (From eqn. (iii))

$\Rightarrow PQ \perp RQ$

Similarly, we can show that

$RQ \perp SR$,

$SR \perp PS$, and

$PS \perp PQ$

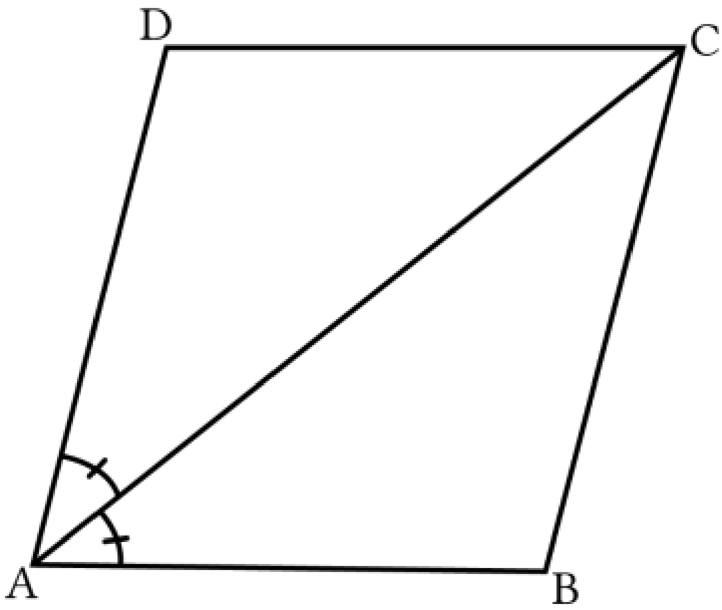
Hence, it is proved that $\square PQRS$ is a square.

Question 6

A diagonal of a parallelogram bisects one of its angles.

Show that it is a rhombus.

Solution:



Let ABCD be the parallelogram. AC bisects $\angle A$.

We need to prove that ABCD is a rhombus.

Since AC bisect $\angle$A,

$\therefore \angle DAC = \angle BAC$ .....(i)

Since $AB \parallel CD$ and AC is transversal,

$\therefore \angle BAC = \angle ACD$......(ii) (Pair of alternate angles)

From (i) and (ii) we have

$\angle DAC = \angle ACD$

In ΔACD,

$\angle DAC = \angle ACD$

$\therefore CD = AD$ .....(iii) (Sides opposite to equal angles are equal)

Since $ABCD$ is a parallelogram,

$AB = CD$ and $BC = AD$ .....(iv) (Opposite sides of parallelogram are equal)

From (iii) and (iv) we have

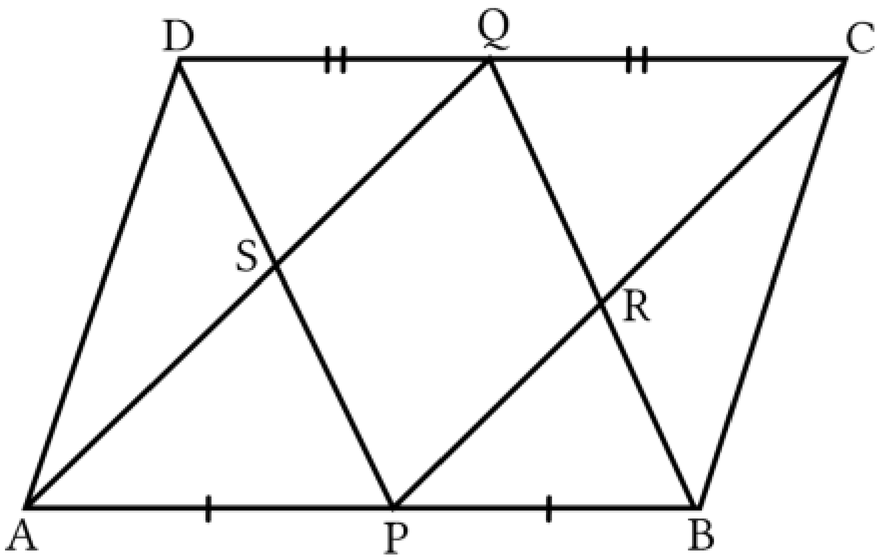
$AB = BC = CD = DA$.

$\therefore$ Parallelogram $ABCD$ is a rhombus.

Question 7

P and Q are the midpoints of the opposite sides AB and CD of a parallelogram ABCD. AQ intersects DP at S and BQ intersects CP at R. Show that PRQS is a parallelogram.

Solution:



It is given that ABCD is a parallelogram,

$\Rightarrow AB \parallel DC$ and $AB = DC$

$\Rightarrow AB \parallel DC$ and $\frac{1}{2}AB = \frac{1}{2}DC$

Since P and Q are the midpoints of AB and CD,

$\Rightarrow AP \parallel CQ$ and $AP = CQ$

$\Rightarrow APCQ$ is a parallelogram.

Then $AQ \parallel PC$ .....(i) (Opposite sides of a parallelogram are parallel).

Similarly, $DPBQ$ is a parallelogram.

$\Rightarrow DP \parallel QB$ .....(ii)

From (i) and (ii), we get

$AQ \parallel PC$ and $DP \parallel QB$

$\Rightarrow SQ \parallel PR$ and $SP \parallel QR$

Hence, it is proved that ▱PRQS is a parallelogram.

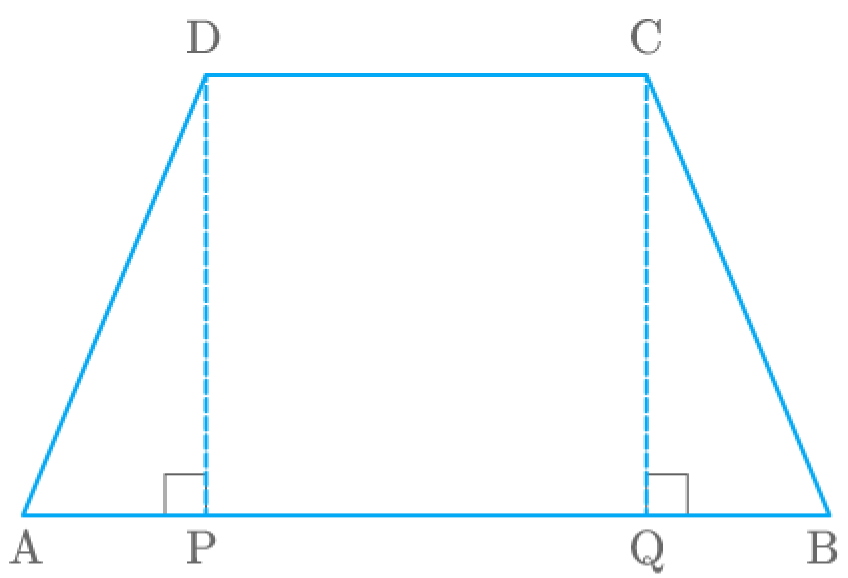
Question 8

ABCD is a quadrilateral in which $AB \parallel DC$ and $AD = BC$.

Prove that $\angle A = \angle B$ and $\angle C = \angle D$.

Solution:

Let us construct $DP \perp AB$ and $CQ \perp AB$



Now, in ΔAPD and ΔBQC,

$\angle APD = \angle BQC = 90°$

$AD = BC$ (Given)

$DP = CQ$ (Distance between || lines)

$\Delta APD \cong \Delta BQC$ (By RHS property)

So,

$\angle A = \angle B \left(C.P.C.T \right)$.

We are given that

$DC \parallel AB$

So, $\angle A + \angle C = \angle B + \angle D$

$\therefore \angle C = \angle D \left[\because \angle A = \angle B \right]$

Hence, it is proved.

Question 9

In Fig. 8.11, AB || DE, AB = DE, AC || DF and AC = DF.

Prove that BC || EF and BC = EF.

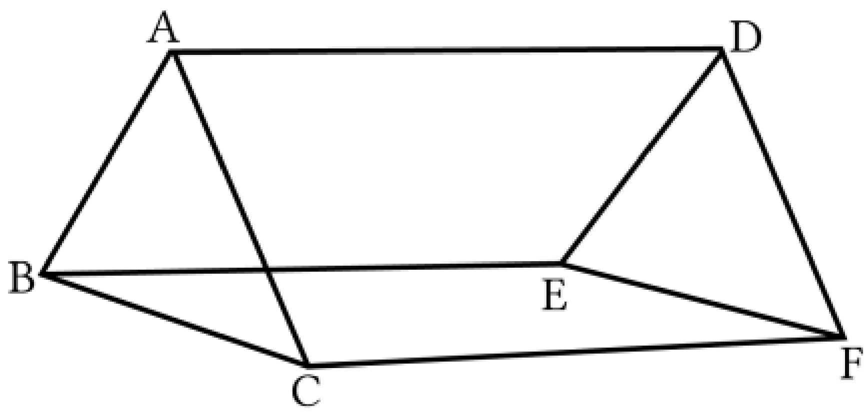


Fig. 8.11

Solution:

It is given that

$AC \parallel DF$ and $AC = DF$,

$\therefore$ $\square ACFD$ is a $\parallel$gram

$\Rightarrow AD \parallel CF \quad .....(i)$

And $AD = CF....(ii)$ (Opposite sides of a $\parallel$gram)

Now, $AB \parallel DE$ and $AB = DE$

$\therefore$ $\square ABED$ is a $\parallel$ gram

$\Rightarrow AD \parallel BE \; ....(iii)$

And $AD = BE \; ......(iv)$ (Opposite sides of a $\parallel$gram)

From (i), (ii), (iii) and (iv), we get

$CF = BE$ and $CF \parallel BE$

$\therefore$ ▱$BCFE$ is $\parallel$ gram

Hence, we can say that $BC \parallel EF$ and $BC = EF$ (Opposite sides of a $\parallel$gram).

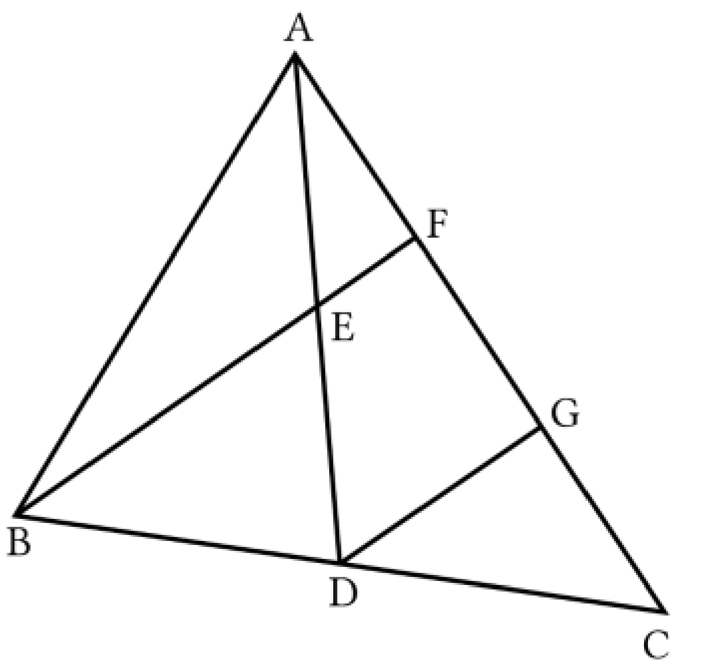
Hence, it is proved.

Question 10

E is the midpoint of a median AD of ABC and BE is produced to meet AC at F.

Show that $AF = \frac{1}{3}AC.$

Solution:



Given: AD is the median of ΔABC and E is the midpoint of AD, also BE meets AC at F.

Let's Draw $DG \parallel BF$

In ΔADG, E is the midpoint of AD and $EF \parallel DG$.

By converse of the midpoint theorem, we have

F is the midpoint of AG .

$\Rightarrow AF = FG \quad .....(i)$

Similarly, in ΔBCF, D is the midpoint of BC and

$DG \parallel BF$.

Therefore, G is the midpoint of CF.

Hence, $FG = GC \quad .....(ii)$

From equations (i) and (ii), we get

$AF = FG = GC \quad .....(iii)$

From the figure we have, $AF + FG + GC = AC$

$\Rightarrow AF + AF + AF = AC$ (From (iii))

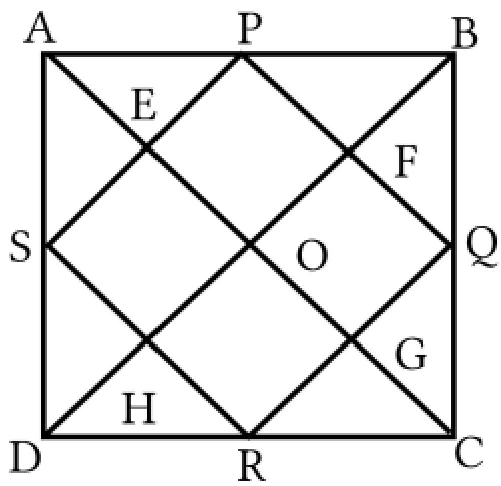
$\Rightarrow 3AF = AC$

Hence, $AF = \frac{AC}{3}$.

Question: 11

Show that the quadrilateral formed by joining the midpoints of the consecutive sides of a square is also a square.

Solution:



Let ABCD be the square such that $AB = BC = CD = DA$.

Also, $AC = BD$ and let P, Q, R and S are the mid points of the sides AB, BC, CD and DA respectively.

In ΔABC,

P and Q are the mid-points of AB and BC respectively.

$\therefore PQ \parallel AC \text{ and } PQ = \frac{1}{2} \times AC$ (By the midpoint theorem) .... (i)

Similarly in ΔADC,

$SR \parallel AC$ and $SR = \frac{1}{2} AC$ (By the mid-point theorem) ... (ii)

Clearly, $PQ \parallel SR$ and $PQ = SR$

Since, in the quadrilateral PQRS, one pair of opposite sides is equal and parallel to each other, it is a parallelogram.

$\therefore PS \parallel QR$ and $PS = QR$ (Opposite sides of a parallelogram are equal and parallel) .... (iii)

In ΔBCD, Q and R are the mid-points of side BC and CD respectively.

$\therefore QR \parallel BD \text{ and } QR = \frac{1}{2} BD$ (By the midpoint theorem) ... (iv)

However, the diagonals of a square are equal,

$AC = BD\ldots \text{ (v)}$

By using equation (i), (ii), (iii), (iv), and (v), we obtain

$PQ = QR = SR = PS$

We know that diagonals of a square are perpendicular bisector of each other.

$\therefore \angle AOB = \angle COD = \angle BOC = \angle DOA = 90°$

Now, in the quadrilateral EOHS, we have

$SE \parallel OH$.

Therefore, $\angle AOD + \angle AES = 180°$ (Corresponding angles)

$\Rightarrow \angle AES = 180° - 90° = 90°$

Again, $\angle AES + \angle SEO = 180°$ (Linear pair)

$\angle SEO = 180° - 90° = 90°$

Similarly $SH \parallel EO$

Therefore, $\angle AOD + \angle DHS = 180°$ (Corresponding angles)

$\angle DHS = 180° - 90° = 90°$

Again, $\angle DHS + \angle SHO = 180°$ (Linear pair)

$\angle SHO = 180° - 90° = 90°$

Again, in the quadrilateral EOHS, we have

$\angle SEO = \angle SHO = \angle EOH = 90°$.

Therefore, by the angle sum property of quadrilateral EHOS, we get $\angle SEO + \angle SHO + \angle EOH + \angle ESH = 360°$

$\Rightarrow 90° + 90° + 90° + ESH = 360°$

$\Rightarrow \angle ESH = 90°$

In the same manner, in quadrilaterals EPFO, FQGO and GOHR, we get

$\angle HRG = \angle FQG = \angle EPF = 90°$

Therefore, in the quadrilateral PQRS, we have

$PQ = QR = SR = PS$ and $ESH = HRG$

$= \angle FQG = \angle EPF = 90°$

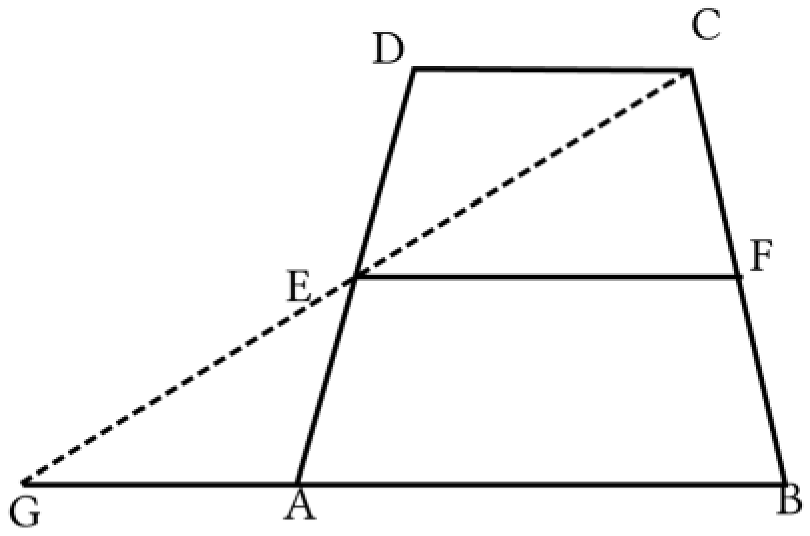
Hence, PQRS is a square.

Question: 12

E and F are respectively the midpoints of the non-parallel sides AD and BC of a trapezium ABCD. Prove that $EF \parallel AB$ **and** $EF = \frac{1}{2}(AB + CD)$.

(Hint: Join CE and produce it to meet AB produced at G.)

Solution:



Let's join CE, which on producing, meet BA produced at G.

In ΔEDC and ΔEAG,

$\angle CED = \angle GEA$ (Vertically opposite angles)

$EA = ED$ ($\because$ E is midpoint of AD)

$\angle EAG = \angle EDC$ (Alternate interior angles are equal as $AB \parallel DC$ and AD is a transversal line)

$\therefore \Delta EAG \cong \Delta EDC$ (By ASA congruency rule)

$\therefore AG = DC$

$CE = EG$

(By CPCT)

In ΔCGB,

E is the midpoint of CG ($\because CE = EG$; as above proved)

F is the midpoint of BC (Given)

$\therefore$ By the midpoint theorem $EF \parallel AB$ and $EF = \frac{1}{2}BG$.

But, $BG = BA + AG = BA + CD$ (As proved above, $AG = CD$)

Hence $EF \parallel AB$ and

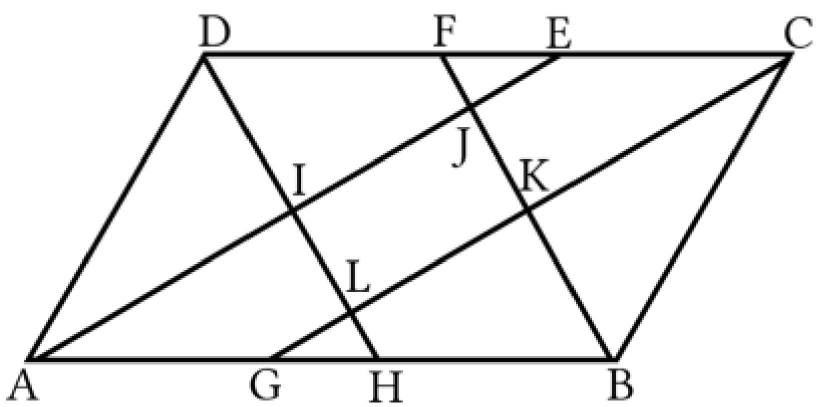
$$EF = \frac{1}{2}(AB + CD).$$

Hence, it is proved.

Question: 13

Prove that the quadrilateral formed by the bisectors of the angles of a parallelogram is a rectangle.

Solution:



Let ABCD is a parallelogram. AE bisects $\angle BAD$, BF bisects $\angle ABC$, CG bisect $\angle BCD$ and DH bisect $\angle ADC$.

We need to prove that LKJI is a rectangle.

Since ABCD is a parallelogram,

$\angle BAD + \angle ABC = 180°$ (Adjacent angles of a parallelogram are supplementary).

$$\frac{1}{2}\angle BAD + \frac{1}{2}\angle ABC = \frac{1}{2}\times 180°$$

But,

$$\angle BAJ = \frac{1}{2}\angle BAD$$

And $\angle ABJ = \frac{1}{2}\angle ABC$ (Given)

$$\Rightarrow \angle BAJ + \angle ABJ = 90°$$

ΔABJ is a right triangle since its acute interior angles are complementary.

$$\Rightarrow \angle AJB = 90° \Rightarrow \angle IJK = 90°$$

Similarly in ΔCDL,

$$\angle CDL + \angle DCL = 90°$$

$$\therefore \angle DLC = 90°$$

In ΔADI and ΔCBK,

$\angle ADI = \angle CBK$ (Opposite angles of a parallelogram are equal)

$AD = BC$ (Opposite sides of a parallelogram are equal)

$\angle DAI = \angle BKC$ (Opposite angles of a parallelogram are equal)

$\therefore \Delta ADI \cong \Delta CBK$ (By ASA congruency rule)

So, $\angle AID = \angle BKC$ (By CPCT)

Now, $\angle AID = \angle JIL$ and $\angle BKC = \angle LKJ$ (Vertical opposite angles)

$\Rightarrow \angle LKJ = \angle JIL$

$\Rightarrow \angle JIL = \angle LKJ = 90^\circ$

Since all four angles of quadrilateral LKJI are right angle, therefore, LKJI is a rectangle.

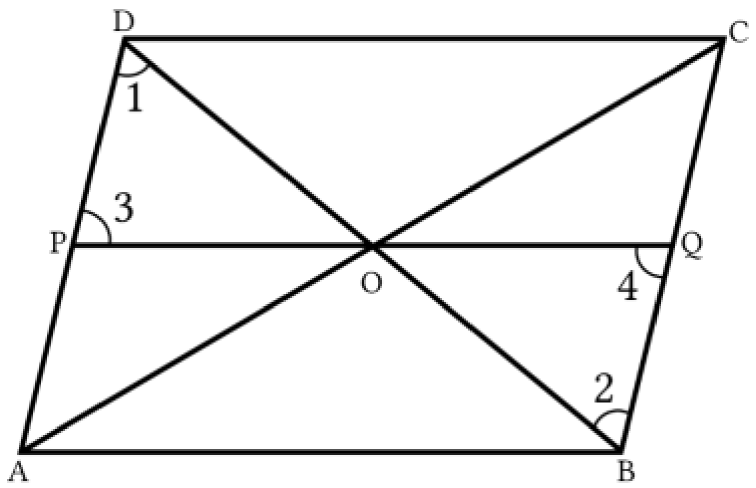
Hence, it is proved.

Question: 14

P and Q are points on the opposite sides AD and BC of a parallelogram ABCD such that PQ passes through the point of intersection O of its diagonals AC and BD.

Show that $OP = OQ$.

Solution:



It is given that $AD \parallel BC$ (Opposite sides of a parallelogram are parallel),

So, $PD \parallel BQ$.

$\therefore \angle 1 = \angle 2$ and $\angle 3 = \angle 4$ (Alternate interior angles are equal)

Now, In ΔDOP and ΔBOQ

$\angle 3 = \angle 4$

$\angle 1 = \angle 2$ (As proved above)

$OD = OB$ (Diagonals of a parallelogram bisect each other).

$\therefore \Delta DOP \cong \Delta BOQ$ (By AAS congruency rule)

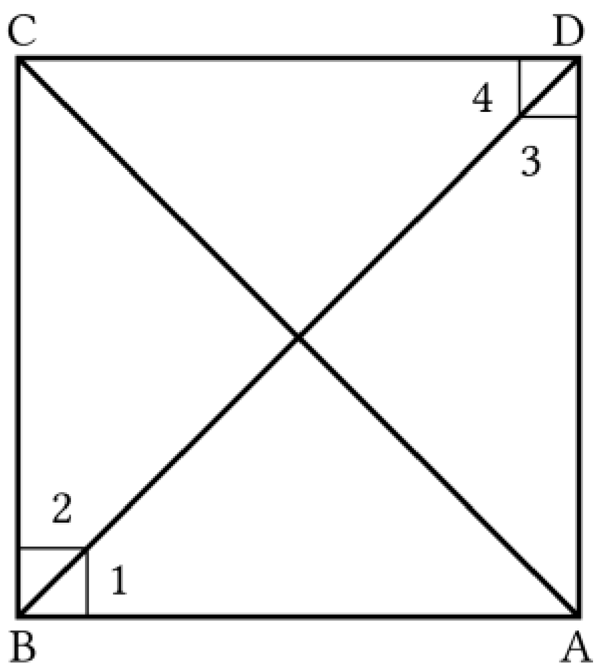
$\therefore OP = OQ$ (By CPCT)

Question: 15

ABCD is a rectangle in which diagonal BD bisects $\angle B$.

Show that ABCD is a square.

Solution:



Let's join AC.

We have $DC \parallel AB$ (Opposite sides of a rectangle), So,

$\angle 4 = \angle 1 ...(\text{i})$

Similarly, $\angle 3 = \angle 2 ....(\text{ii})$

$\angle 1 = \angle 2 .....(\text{iii})$ (given)

From (i), (ii), (iii), we get

$\angle 3 = \angle 4$

In ΔBDA and ΔBDC

$\angle 1 = \angle 2$ (Given)

$BD = BD$ (Common)

$\angle 3 = \angle 4$ (As proved above)

$\therefore \Delta BDA \cong \Delta DBC$ (By ASA congruency rule)

$AD = CD$

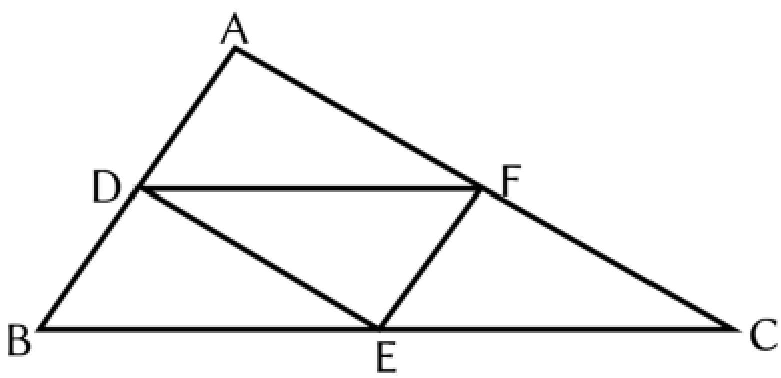
$AB = CB$ (CPCT)

$\therefore$ Rectangle ABCD is a square.

Question: 16

D, E and F are respectively the midpoints of the sides AB, BC and CA of a triangle ABC. Prove that by joining these midpoints D, E and F, the triangle ABC is divided into four congruent triangles.

Solution:



In ΔABC,

DF joins the midpoints of the sides AB and AC respectively.

Then, $DF \parallel BC \text{ and } DF = \frac{1}{2}BC = BE = EC$

(Using the midpoint theorem)

Similarly, DE joins midpoints of AB and BC respectively.

$\therefore\ DE \parallel AC \text{ and } DE = \frac{1}{2}AC = AF = FC$

Also, EF joins the midpoints of BC and AC respectively.

$\therefore \mathrm{EF} \parallel \mathrm{AB}$ and $\mathrm{EF} = \frac{1}{2}\mathrm{AB} = \mathrm{AD} = \mathrm{DB}$

Now, in ΔBDE and ΔDEF

$\mathrm{BD} = \mathrm{EF}$ (As proved above)

$\mathrm{DE} = \mathrm{DE}$ (Common)

$\mathrm{BE} = \mathrm{DF}$ (Given)

So, by SSS rule,

$\Delta\mathrm{BDE} \cong \Delta\mathrm{FED}.$

Similarly,

$\Delta\mathrm{EFC} \cong \mathrm{FED}$

And,

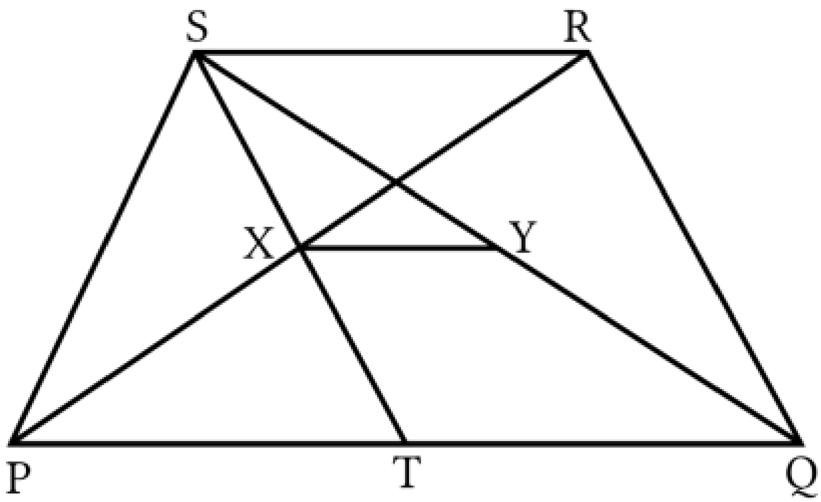
$\Delta\mathrm{EFD} \cong \Delta\mathrm{ADF}$

So, all 4 triangles are congruent.

Question: 17

Prove that the line joining the mid-points of the diagonals of a trapezium is parallel to the parallel sides of the trapezium.

Solution:



Let PQRS be a trapezium in which $PQ \parallel RS$ and let X and Y be the midpoints of the diagonals PR and SQ, respectively.

We need to prove that

$XY \parallel SR \parallel PQ$.

Now, let us join SX and produce it to intersect PQ produced at T.

In ΔSXR and ΔPXT, we have

$\angle SRX = \angle TPX$ (Alternate interior angles) ,

$PX = XR$ (Given)

and $\angle SXR = \angle PXT$ (Vertically opposite angles)

$\therefore \Delta SXR \cong \Delta TXP$ (By ASA congruence)

$\therefore SR = PT$ and $SX = XT$ (by CPCT)

Thus, in ΔSTQ, the points X and Y are the midpoints ST and SQ respectively.

$\therefore XY \parallel TQ$ (By the converse of mid-point theorem)

$\Rightarrow XY \parallel PQ \parallel SR$

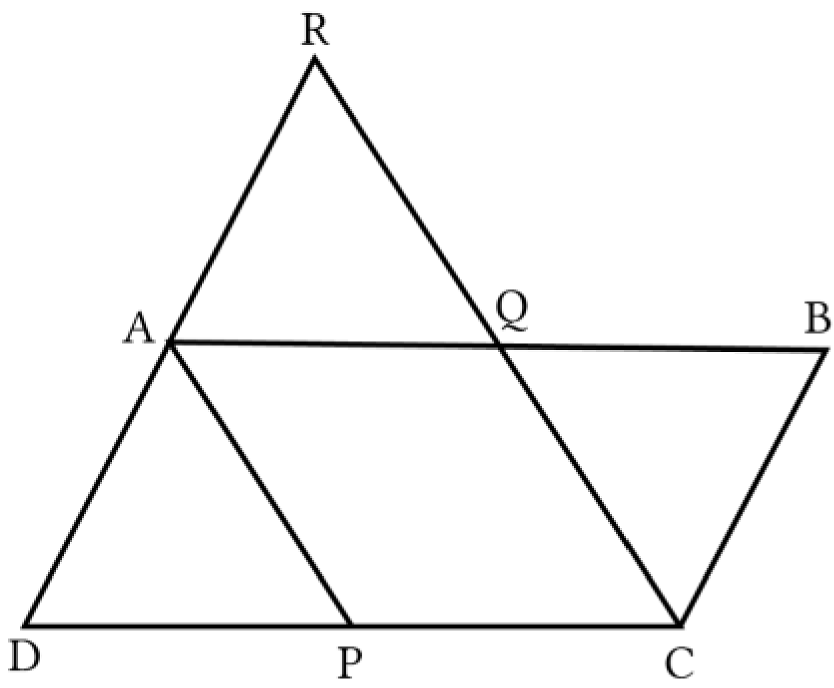
Hence, it is proved.

Question: 18

P is the midpoint of the side CD of a parallelogram ABCD. A line through C parallel to PA intersects AB at Q and DA produced at R. Prove that $DA = AR$ and

$CQ = QR$.

Solution:



In ΔCDR, P is midpoint DR and $AP \parallel CR$

By the converse of the midpoint theorem, we have

$DA = AR$

But, $AD = BC$ (Opposite sides of a parallelogram are equal)

$\Rightarrow AR = BC$.......(i)

Again, in ΔCDR,

A is midpoint of DR and $AQ \parallel CD$.

$\because$ ABCD is a parallelogram and its opposite sides are parallel to each other

$\therefore Q$ Midpoint of CR (By the midpoint theorem)

$\Rightarrow CQ = QR$

Hence, it is proved.