NCERT Solution

Class 9 Maths

Chapter 7- Triangles

Exercise 7.1 (8)

Question: 1

In quadrilateral ACBD, $AC = AD$ and AB bisects $\angle A$ (See Fig. 7.16). Show that $\Delta ABC \cong \Delta ABD$. What can you say about BC and BD?

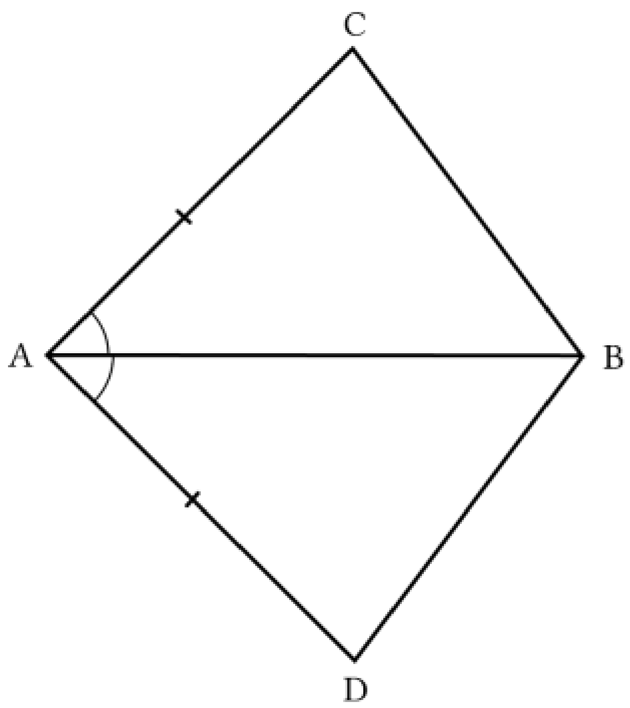


Fig. 7.16

Solution:

Given:

In quadrilateral ACBD,

$AC = AD$ and AB bisects $\angle A$

We need to prove that $\Delta ABC \cong \Delta ABD$

Now, in ΔABC and ΔABD,

$AB = AB$ (Common)

$\angle CAB = \angle DAB$ (AB is the angle bisector)

$AC = AD$ (Given)

Therefore, $\Delta ABC \cong \Delta ABD$ by SAS congruence condition.

$BC = BD$ (By CPCT)

Question: 2

ABCD is a quadrilateral in which AD=BC and

$\angle DAB = \angle CBA$ **(see Fig 7.17). Prove that**

(i) $\Delta ABD \cong \Delta BAC$

(ii) $BD = AC$

(iii) $\angle ABD = \angle BAC$.

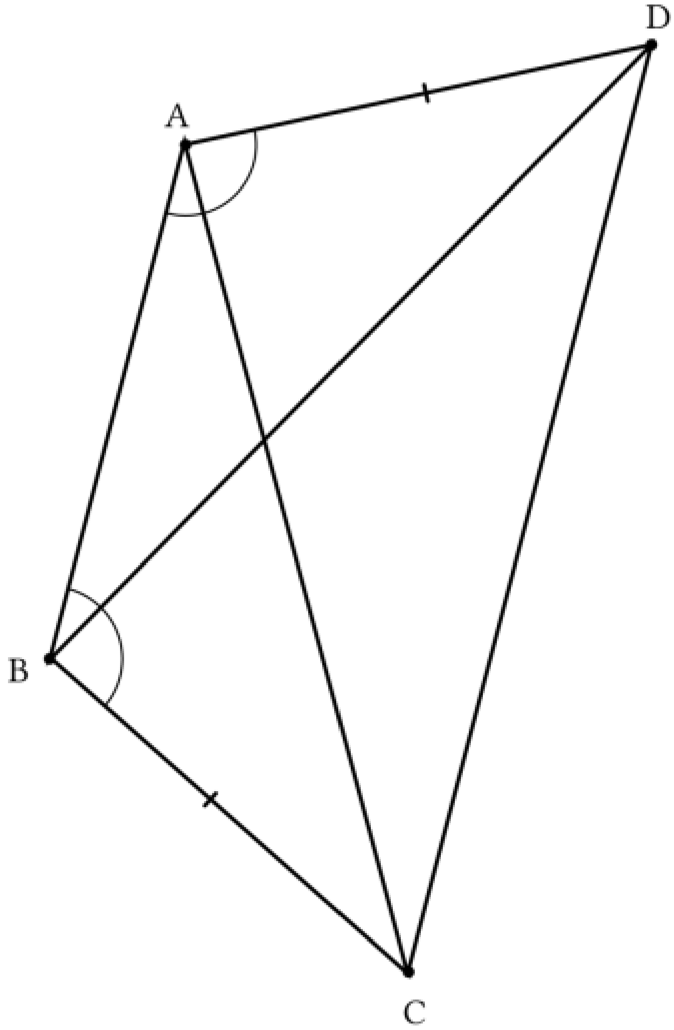


Fig. 7.17

Solution:

We are given:

$AD = BC$

$\angle DAB = \angle CBA$

(i)In ΔABD and ΔBAC,

$AB = BA$ (Common)

$\angle DAB = \angle CBA$ (Given)

$AD = BC$ (Given)

Therefore, $\Delta ABD \cong \Delta BAC$ by SAS congruence criterion.

(ii) Since, $\Delta ABD \cong \Delta BAC$

Therefore, BD=AC(By CPCT)

(iii) Since, $\Delta ABD \cong \Delta BAC$

Therefore, $\angle ABD = \angle BAC$ (By CPCT)

Question: 3

AD and BC are equal perpendiculars to a line segment AB (see Fig. 7.18). Show that CD bisects AB.

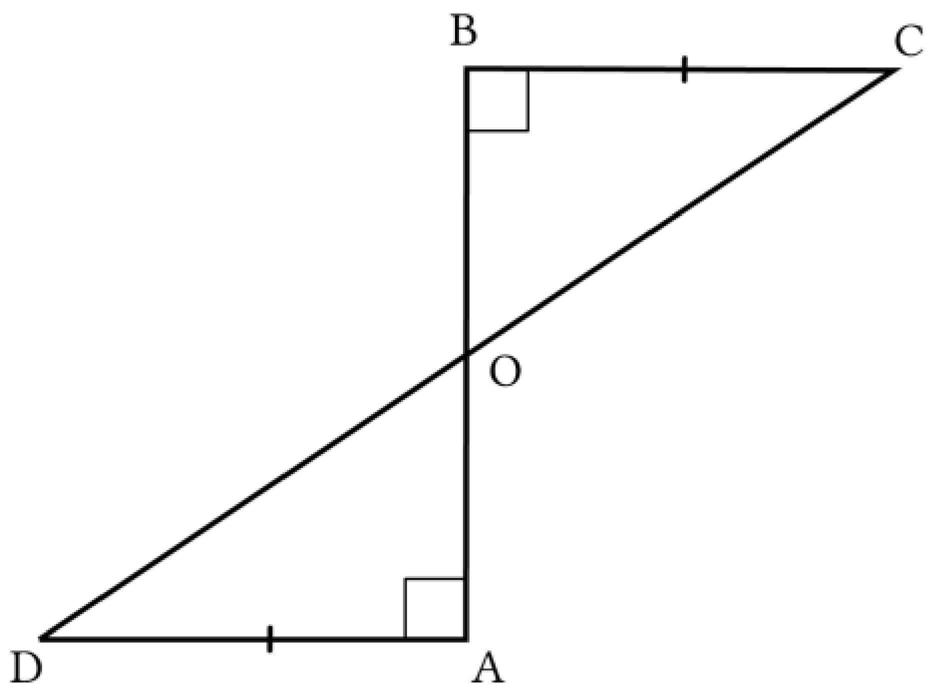


Fig. 7.18

Solution:

Given:

$AD \perp AB$ and $BC \perp AB$.

To Prove: $OA = OB$

Proof:

In ΔAOD and ΔBOC,

$\angle A = \angle B$ (As $AD \perp AB$ and $BC \perp AB$)

$\angle AOD = \angle BOC$ (Vertically opposite angles)

AD=BC (Given)

Therefore, $\Delta AOD \cong \Delta BOC$ (By AAS congruence criterion)

This implies,

$AO = OB$ (By CPCT)

or CD bisects AB.

Question: 4

l and m are two parallel lines intersected by another pair of parallel lines p and q (See Fig. 7.19). Show that $\Delta ABC \cong \Delta CDA$.

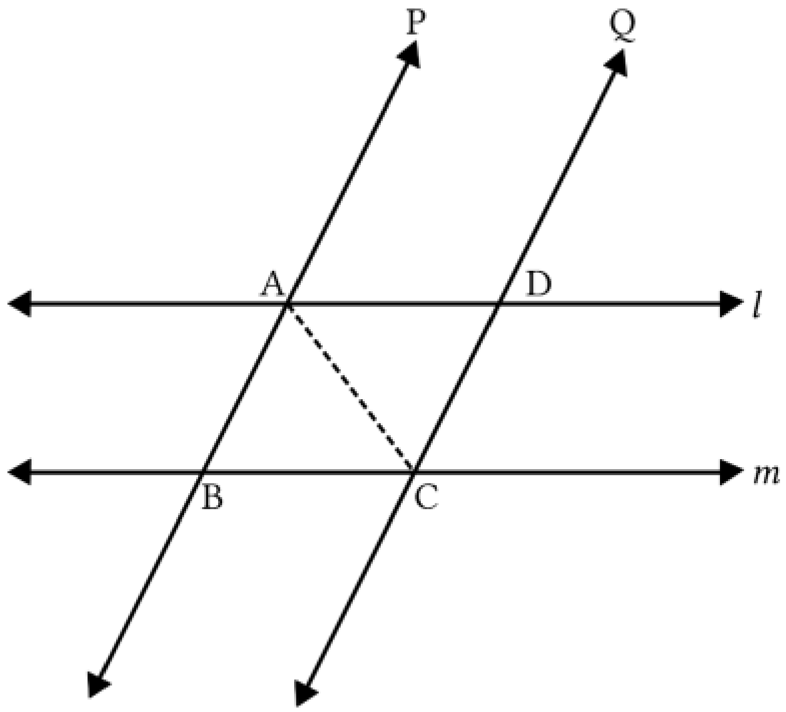


Fig. 7.19

Solution:

Given:

$l \parallel m$ and $p \parallel q$

To prove:

$\Delta ABC \cong \Delta CDA$

Proof:

In ΔABC and ΔCDA,

$\angle BCA = \angle DAC$ (Alternate interior angles)

$AC = CA$ (Common)

$\angle BAC = \angle DCA$ (Alternate interior angles)

Therefore, $\Delta ABC \cong \Delta CDA$ (By ASA congruence condition.)

Question: 5

Line l is the bisector of angle $\angle A$ and B is any point on l.

BP and BQ are perpendiculars from B to the arms of $\angle A$

(see Fig. 7.20). Show that:

(i) $\Delta APB \cong \Delta AQB$

(ii)BP=BQ or B is equidistant from the arms of $\angle A$.

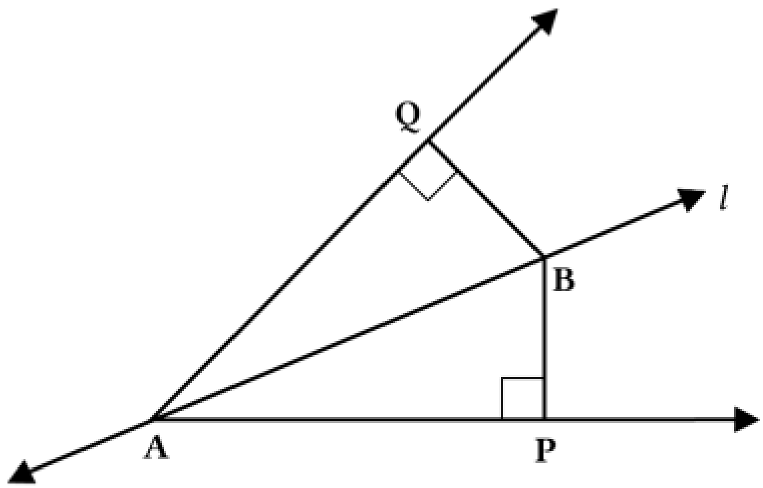


Fig. 7.20

Solution:

Given:

l is the bisector of angle $\angle A$.

BP and BQ are perpendiculars on arms of $\angle A$.

(i) In ΔAPB and ΔAQB,

$\angle P = \angle Q$ (Right angles)

$\angle BAP = \angle BAQ$ (l is the bisector)

$AB = AB$ (Common)

Therefore, $\Delta APB \cong \Delta AQB$ by AAS congruence condition.

(ii) $BP = BQ$ by CPCT. Therefore, B is equidistant from the arms of $\angle A$.

Question: 6

In Fig. 7.21, $AC = AE$, $AB = AD$ and $\angle BAD = \angle EAC$. Show that $BC = DE$.

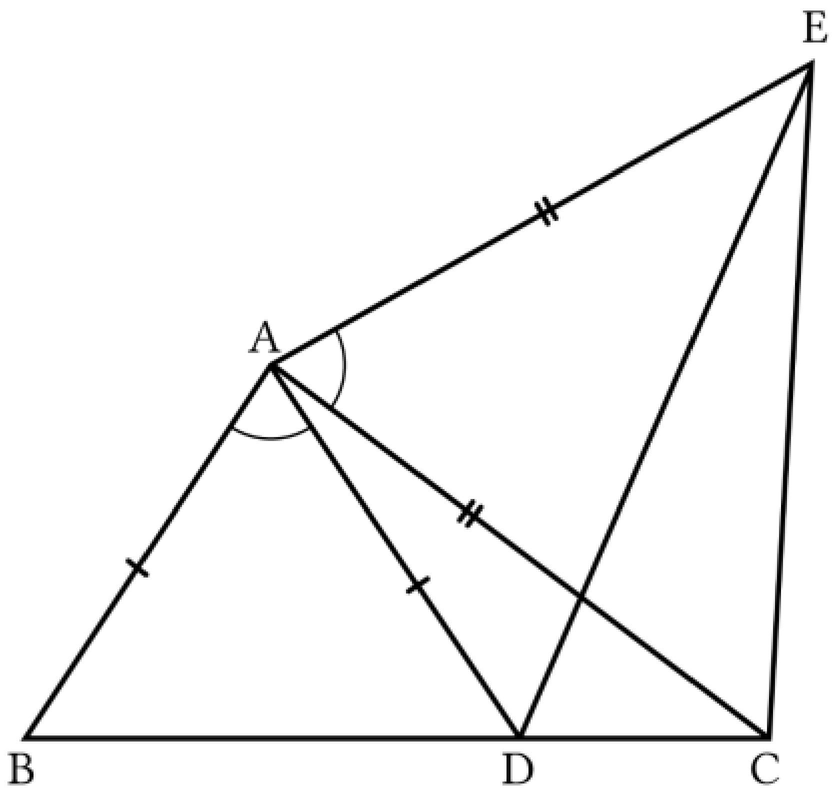


Fig. 7.21

Solution:

Given:

$AC = AE,\ AB = AD$ and $\angle BAD = \angle EAC$

We need to prove that $BC = DE$

Now,

$\angle BAD = \angle EAC$ (Given)

Adding $\angle DAC$ on both the sides, we get

$\angle BAD + \angle DAC = \angle EAC + \angle DAC$

$\Rightarrow \angle BAC = \angle EAD$

In ΔABC and ΔADE,

$AC = AE$ (Given)

$\angle BAC = \angle EAD$ (As proved above)

$AB = AD$ (Given)

Therefore,

$\Delta ABC \cong \Delta ADE$ by SAS congruence condition.

$BC = DE$ (By CPCT).

Question: 7

AB is a line segment and P is its midpoint. D and E are points on the same side of AB such that $\angle BAD = \angle ABE$ and $\angle EPA = \angle DPB$ (See Fig. 7.22). Show that:

(i) $\Delta DAP \cong \Delta EBP$

(ii) $AD = BE$

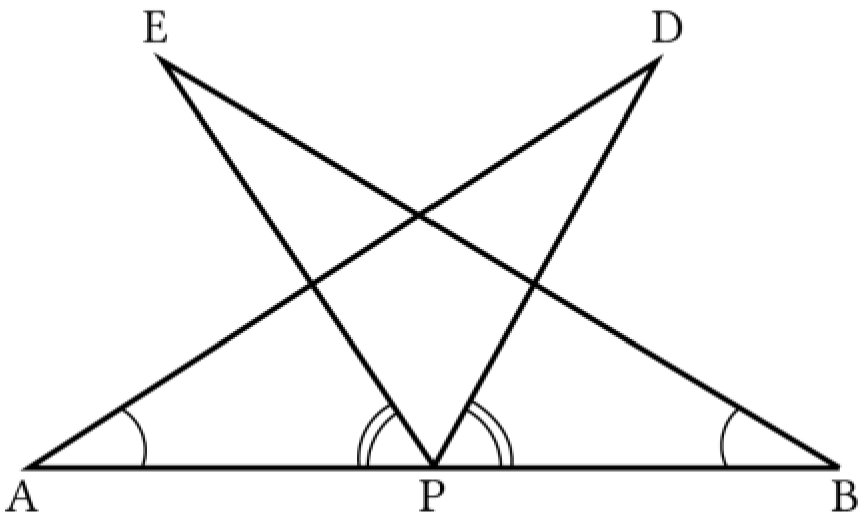


Fig. 7.22

Solution:

Given:

P is the midpoint of AB. i.e., $AP = PB$.

$\angle BAD = \angle ABE$ and $\angle EPA = \angle DPB$...(i)

Since, $\angle EPA = \angle DPB$

Adding $\angle DPE$ both the sides

$\angle EPA + \angle DPE = \angle DPB + \angle DPE$

$\Rightarrow \angle DPA = \angle EPB$

Now, In ΔDAP and ΔEBP, $\angle DPA = \angle EPB$

(As proved above)

$AP = BP$ (Given)

$\angle BAD = \angle ABE$ (Given)

Therefore,

$\Delta DAP \cong \Delta EBP$ by ASA Congruence condition.

(ii) $AD = BE$ by CPCT.

Question: 8

In right triangle ABC, right angled at C, M is the midpoint of hypotenuse AB.C is joined to M and produced to a point D such that $DM = CM$. Point D is joined to point B. Show that:

(i) $\Delta AMC \cong \Delta BMD$

(ii) $\angle DBC$ is a right angle.

(iii) $\Delta DBC \cong \Delta ACB$

(iv) $CM = \frac{1}{2} AB$

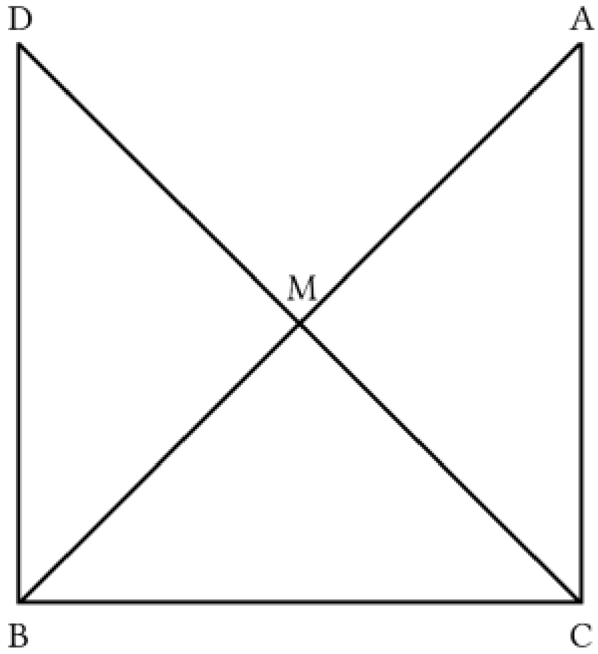


Fig. 7.23

Solution:

Given,

$\angle C = 90°$, M is the midpoint of AB i.e., $AM = MB$ and

$DM = CM$

(i) In ΔAMC and ΔBMD,

$AM = BM$ (Given)

$\angle CMA = \angle DMB$ (Vertically opposite angles)

$CM = DM$ (Given)

Therefore, $\Delta AMC \cong \Delta BMD$ by SAS congruence condition.

$AC = BD \quad \ldots..(\text{i})$ (By CPCT)

(ii) $\angle ACM = \angle BDM$ (By CPCT)

Therefore, $AC \parallel BD$ as alternate interior angles are equal.

Now,

$\angle ACB + \angle DBC = 180°$ (Co-interiors angles)

$\Rightarrow 90° + \angle DBC = 180°$

$\Rightarrow \angle DBC = 90°$

(iii) In ΔDBC and ΔACB,

BC=CB (Common)

$\angle ACB = \angle DBC$ (Right angles)

$DB = AC$ (Using (i))

Therefore,

$\Delta DBC \cong \Delta ACB$ by SAS congruence condition.

$DC = AB \ldots(\text{ii})$ (By CPCT)

(iv)DC=AB (Using (ii))

$\Rightarrow DM + CM = AB$

$\Rightarrow CM + CM = AB \left(\text{As } DM = CM\right)$

$\Rightarrow 2CM = AB$

$\Rightarrow CM = \dfrac{AB}{2}$

Exercise 7.2 (8)

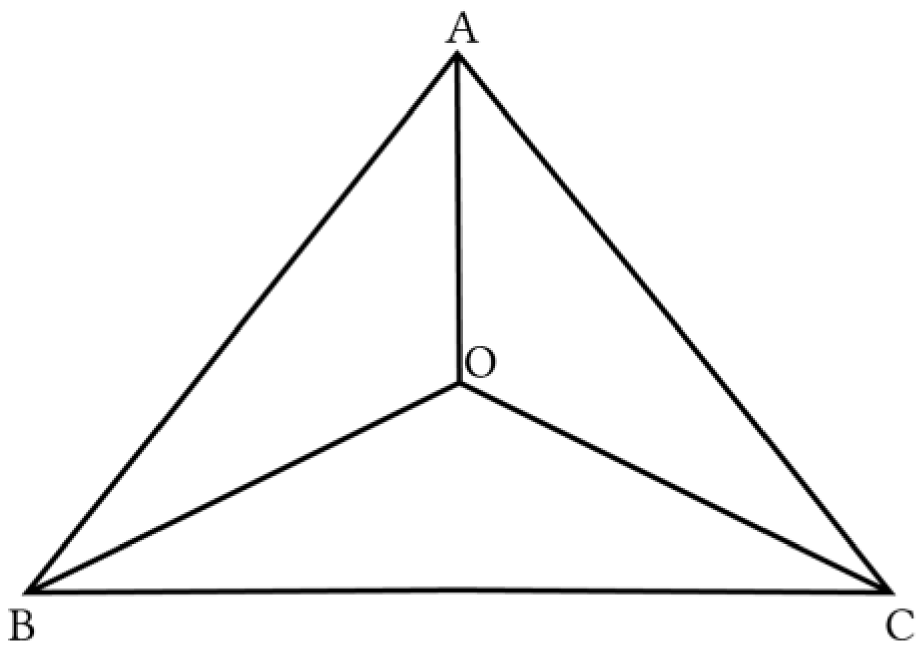
Question: 1

In an isosceles triangle ABC, with $AB = AC,$ the bisectors of $\angle B$ and $\angle C$ intersect each other at O. Join A to O.

Show that:

(i) $OB = OC$

(ii) AO bisects $\angle A$



Solution:

Given,

$AB = AC$, the bisectors of $\angle B$ and $\angle C$ intersect each other at O.

(i) Since, ABC is an isosceles with $AB = AC$,

$\therefore \angle B = \angle C$

$\Rightarrow \frac{1}{2}\angle B = \frac{1}{2}\angle C$ (As OB and OC are angle bisectors)

$\Rightarrow \angle OBC = \angle OCB$

$\Rightarrow OB = OC$ (Side opposite to the equal angles are equal.)

(ii) In ΔAOB and ΔAOC,

$AB = AC$ (Given)

$AO = AO$ (Common)

$OB = OC$ (As above proved)

Therefore, $\Delta AOB \cong \Delta AOC$ by SSS congruence condition.

$\angle BAO = \angle CAO$ By CPCT

Thus, AO bisects $\angle A$

Question: 2

In $\Delta ABC, AD$ is the perpendicular bisector of BC (see Fig. 7.30). Show that ΔABC is an isosceles triangle in which $AB = AC$.

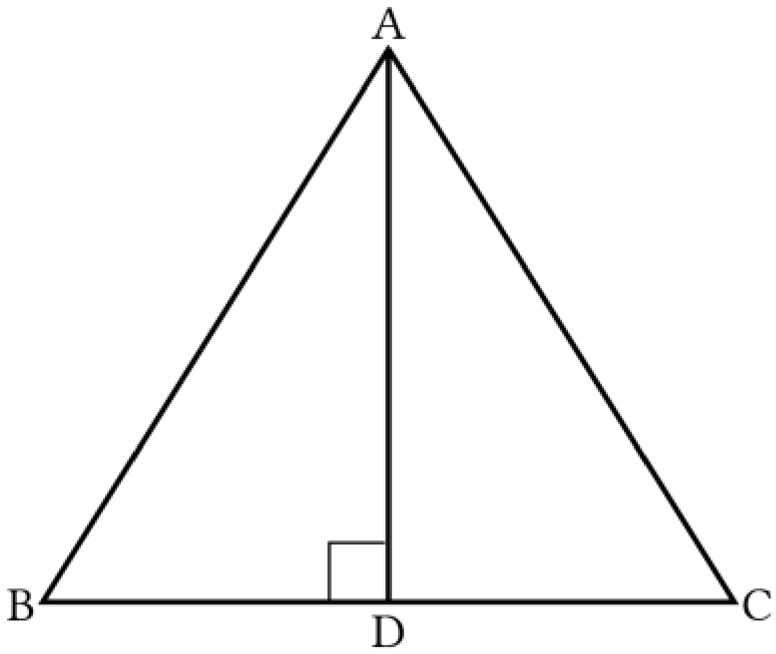


Fig. 7.30

Solution:

Given,

AD is the perpendicular bisector of BC.

To show,

$AB = AC$

Proof,

In ΔADB and ΔADC,

$AD = AD$ (Common)

$\angle ADB = \angle ADC$

$BD = CD$ (AD is the perpendicular bisector)

Therefore, $\Delta ADB \cong \Delta ADC$ by SAS congruence condition.

AB=AC (By CPCT).

Question: 3

ΔABC is an isosceles triangle in which altitudes BE and CF are drawn to equal sides AC and AB respectively (see Fig. 7.31). Show that these altitudes are equal.

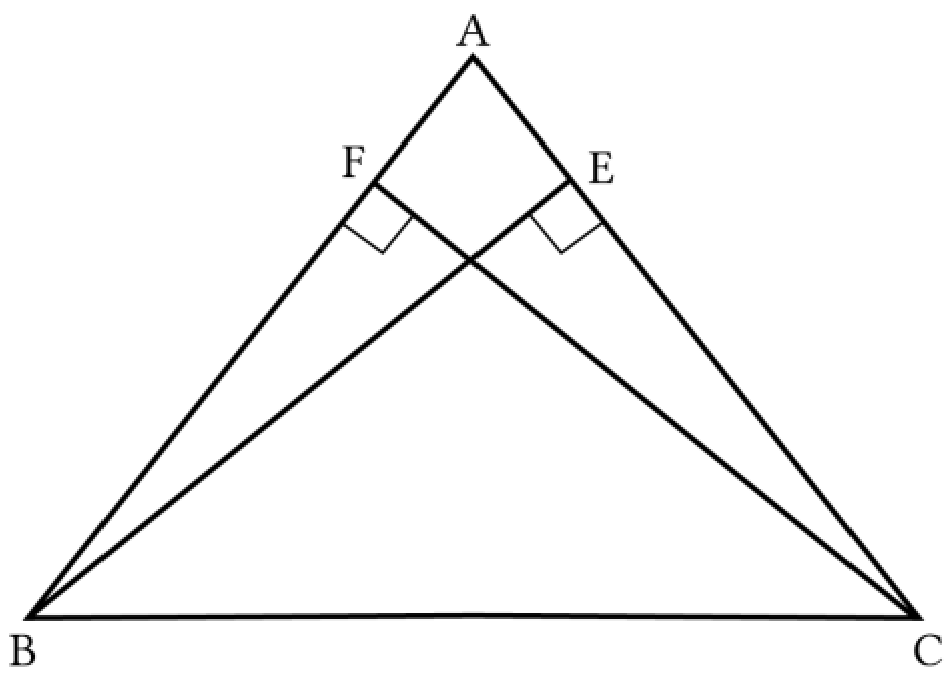


Fig. 7.31

Solution:

Given, in ΔABC

BE and CF are altitudes.

$AC = AB$

To show,

BE=CF

Proof:

In ΔAEB and ΔAFC,

$\angle A = \angle A$ (Common)

$\angle AEB = \angle AFC$ (Right angles)

$AB = AC$ (Given)

Therefore, $\Delta AEB \cong \Delta AFC$ by AAS congruence condition.

Thus, $BE = CF$ by CPCT.

Question: 4

ΔABC is a triangle in which altitudes BE and CF to sides AC and AB are equal (See Fig. 7.32). Show that:

(i) $\Delta ABE \cong \Delta ACF$

(ii) $AB = AC$,i.e., ABC is an isosceles triangle.

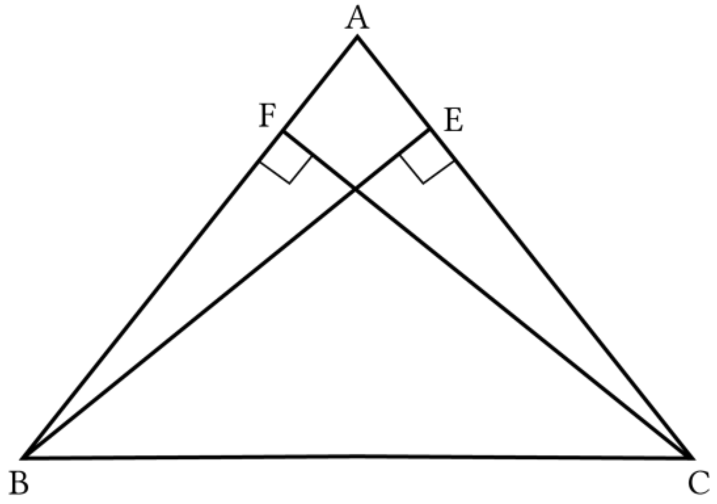


Fig. 7.32

Solution:

Given,

In ΔABC, BE & CF are two altitudes such that,

$BE = CF$

(i) In ΔABE and ΔACF,

$\angle A = \angle A$ (Common)

$\angle AEB = \angle AFC$ (Right angles)

BE=CF(Given)

Therefore, $\Delta ABE \cong \Delta ACF$ by AAS congruence condition.

(ii) Thus, $AB = AC$ by CPCT and therefore ABC is an isosceles triangle.

Question: 5

ΔABC and ΔDBC **are two isosceles triangles on the same base BC (See Fig. 7.33). Show that** $\angle ABD = \angle ACD$.

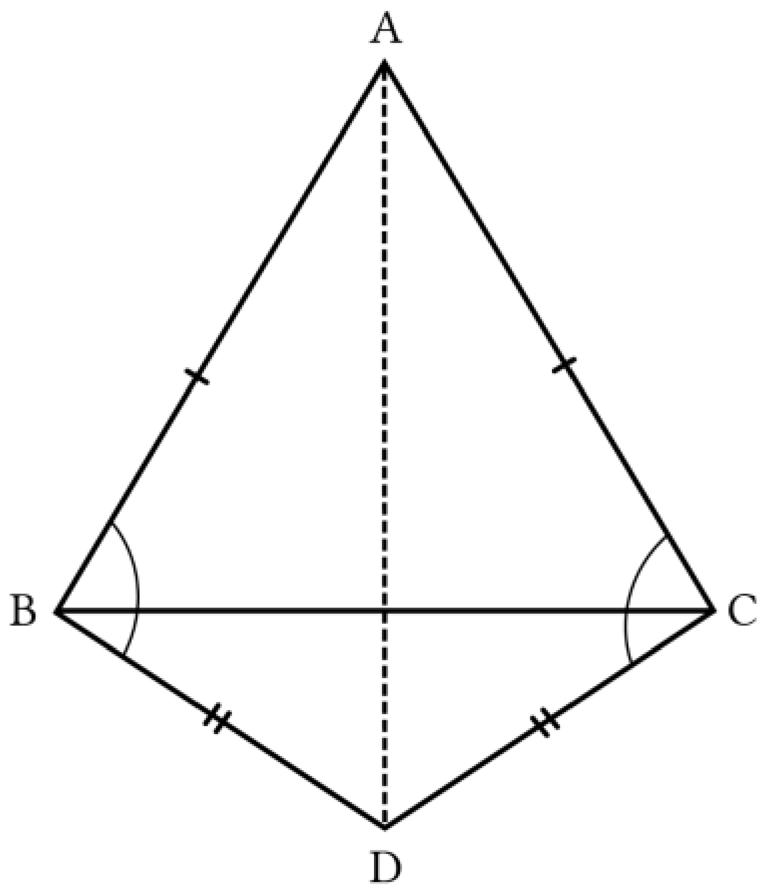


Fig. 7.33

Solution:

Given,

ΔABC and ΔDBC are two isosceles triangles.

To show: $\angle ABD = \angle ACD$.

Proof:

Let us join AD

In ΔABD and ΔACD,

$AD = AD$ (Common)

$AB = AC$ (ΔABC is an isosceles triangle.)

$BD = CD$ (ΔBCD is an isosceles triangle.)

Therefore,

$\Delta ABD \cong \Delta ACD$ (By SSS congruence condition).

Thus, $\angle ABD = \angle ACD$ by CPCT.

Question: 6

ΔABC is an isosceles triangle in which $AB = AC$.

Side BA is produced to D such that $AD = AB$ (See Fig. 7.34). Show that $\angle BCD$ is a right angle.

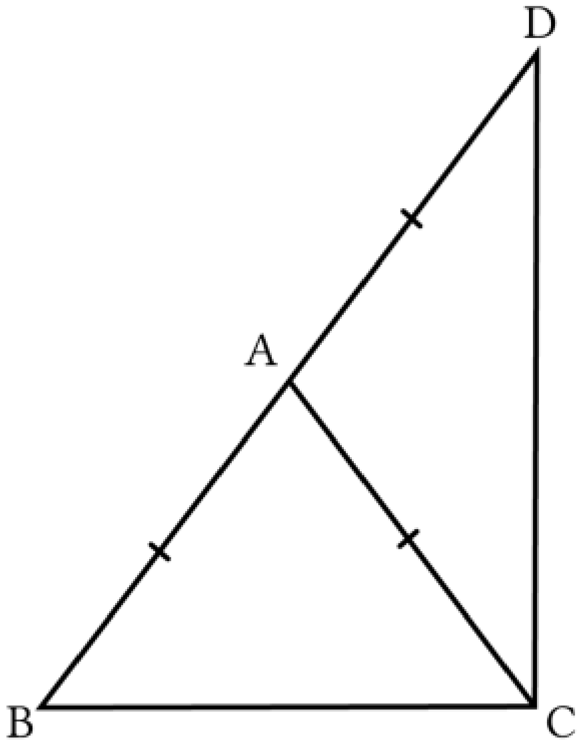


Fig. 7.34

Solution:

Given,

$AB = AC$ and $AD = AB$

$\Rightarrow AC = AD$

To show:

$\angle BCD$ is a right angle.

Proof:

In ΔABC,

$AB = AC$ (Given)

$\Rightarrow \angle ACB = \angle ABC \ldots (i)$(Angles opposite to the equal sides are equal.)

In ΔACD,

$AC = AD$ (Given)

$\Rightarrow \angle CDA = \angle DCA \ldots (ii)$(Angles opposite to the equal sides are equal.)

Now,

In ΔBCD,

$\angle DBC + \angle BCD + \angle CDB = 180°$

$\Rightarrow \angle ABC + \angle BCD + \angle CDA = 180°$

$\Rightarrow \angle ACB + \angle BCD + \angle DCA = 180°$ (From (i) and (ii))

$\Rightarrow (\angle ACB + \angle DCA) + \angle BCD = 180°$

$\Rightarrow \angle BCD + \angle BCD = 180°$

$\Rightarrow 2\angle BCD = 180°$

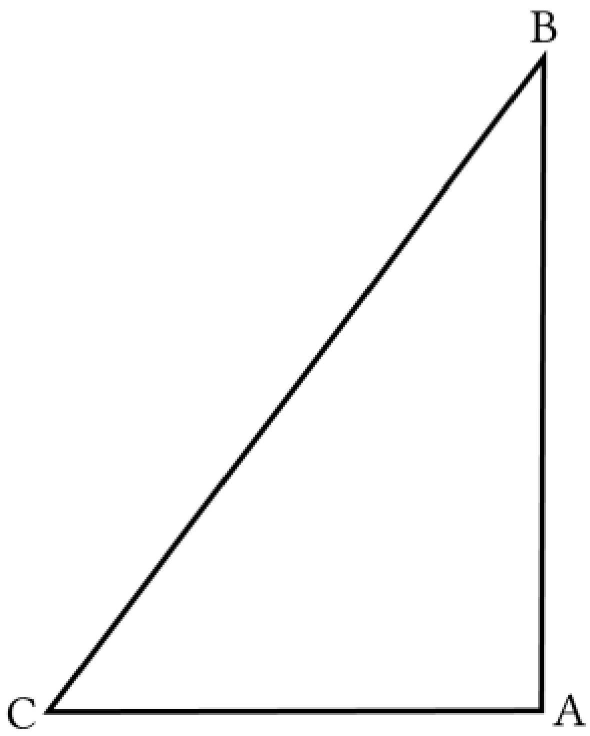
$\Rightarrow \angle BCD = 90°$

Hence, it is proved.

Question: 7

ABC is a right angled triangle in which $\angle A = 90°$ and $AB = AC$. Find $\angle B$ and $\angle C$

Solution:



Given,

$\angle A = 90°$ and $AB = AC$

$\Rightarrow \angle B = \angle C$ (Angles opposite to the equal sides are equal.)

Now,

$\angle A + \angle B + \angle C = 180°$ (Sum of the interior angles of a triangle is 180°.)

$\Rightarrow 90° + 2\ \angle B = 180°$

$\Rightarrow 2\ \angle B = 90°$

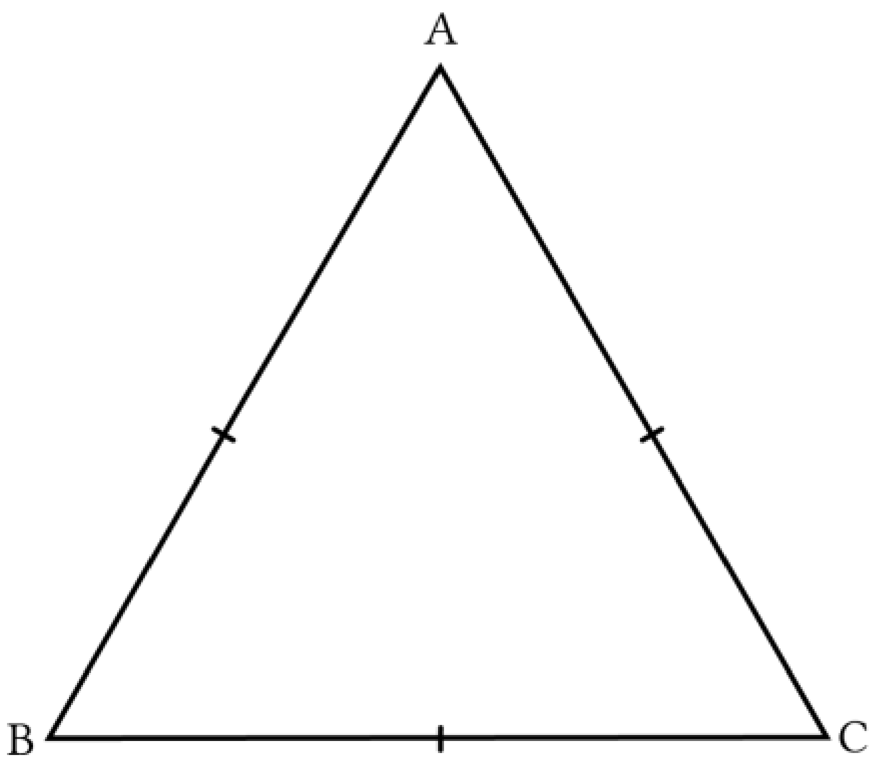
$\Rightarrow \angle B = 45°$

Thus, $\angle B = \angle C = 45°$

Question: 8

Show that the angles of an equilateral triangle are 60° each.

Solution:



Let ABC be an equilateral triangle.

$BC = AC = AB$ (Length of all sides is same)

$\Rightarrow \angle A = \angle B = \angle C$ (Angles opposite to the equal sides are equal.)

Also,

$\angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow 3\angle A = 180^\circ$

$\Rightarrow \angle A = 60^\circ$

Therefore, $\angle A = \angle B = \angle C = 60^\circ$

Thus, the angles of an equilateral triangle are 60° each.

Exercise 7.3 (5)

Question: 1

ΔABC and ΔDBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (see Fig. 7.39). If AD is extended to BC such that it intersects BC at P, show that:

(i) $\Delta ABD \cong \Delta ACD$

(ii) $\Delta ABP \cong \Delta ACP$

(iii) AP bisects $\angle A$ as well as $\angle D$.

(iv) AP is the perpendicular bisector of BC.

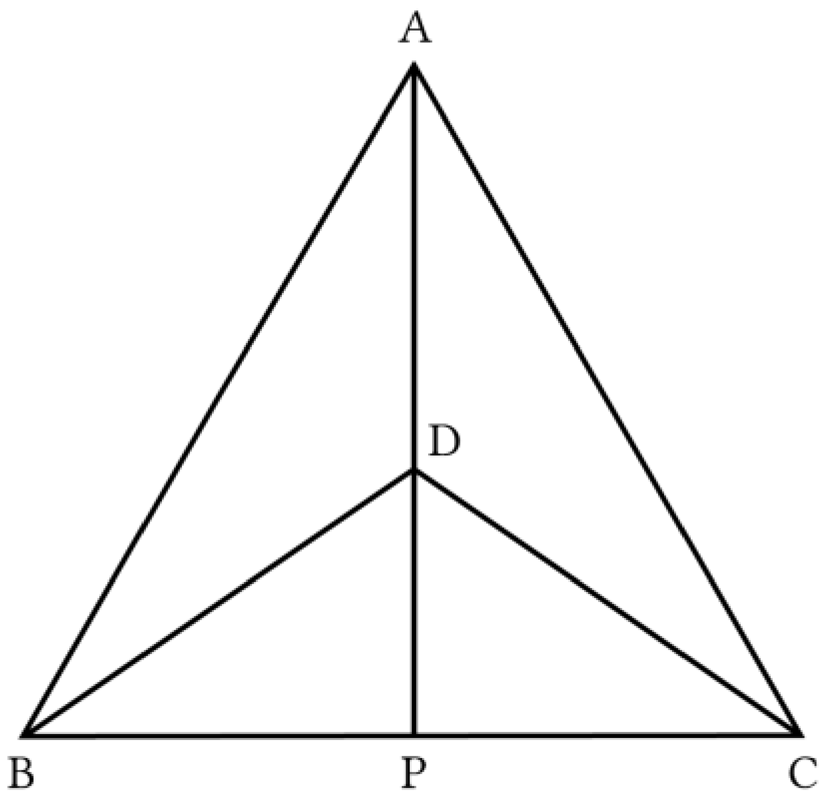


Fig. 7.39

Solution:

Given,

ΔABC and ΔDBC are two isosceles triangles.

(i) In ΔABD and ΔACD,

AD=AD (Common)

AB=AC (ΔABC is isosceles)

BD=CD (ΔDBC is isosceles)

Therefore, $\Delta ABD \cong \Delta ACD$ by SSS congruence condition.

(ii) In ΔABP and ΔACP,

AP=AP (Common) $\angle PAB = \angle PAC$

[$\because \Delta ABD \cong \Delta ACD$ So, by CPCT]

$AB = AC$ (ΔABC is isosceles)

Therefore,

$\Delta ABP \cong \Delta ACP$ by SAS congruence condition.

(iii) $\angle PAB = \angle PAC$ by CPCT as $\Delta ABD \cong \Delta ACD$.

Therefore, AP bisects $\angle A$. $---(1)$

Also,

In ΔBPD and ΔCPD,

PD=PD (Common)

BD=CD (ΔDBC is isosceles.)

$BP = CP$ ($\because \Delta ABP \cong \Delta ACP$ so by CPCT.)

Therefore,

$\Delta BPD \cong \Delta CPD$ by SSS congruence condition.

Thus, $\angle BDP = \angle CDP$ by CPCT. $---(2)$

By(1) and (2), we can say that AP bisects $\angle A$ as well as $\angle D$

.

(iv) $\angle BPD = \angle CPD$ (By CPCT as $\Delta BPD \cong \Delta CPD$)

$BP = CP$

(Proved above)

Also,

$\angle BPD + \angle CPD = 180^\circ$ (BC is a straight line.)

$\Rightarrow 2\angle BPD = 180^\circ$

$\Rightarrow \angle BPD = 90^\circ$

Therefore,

AP is the perpendicular bisector of BC.

Question: 2

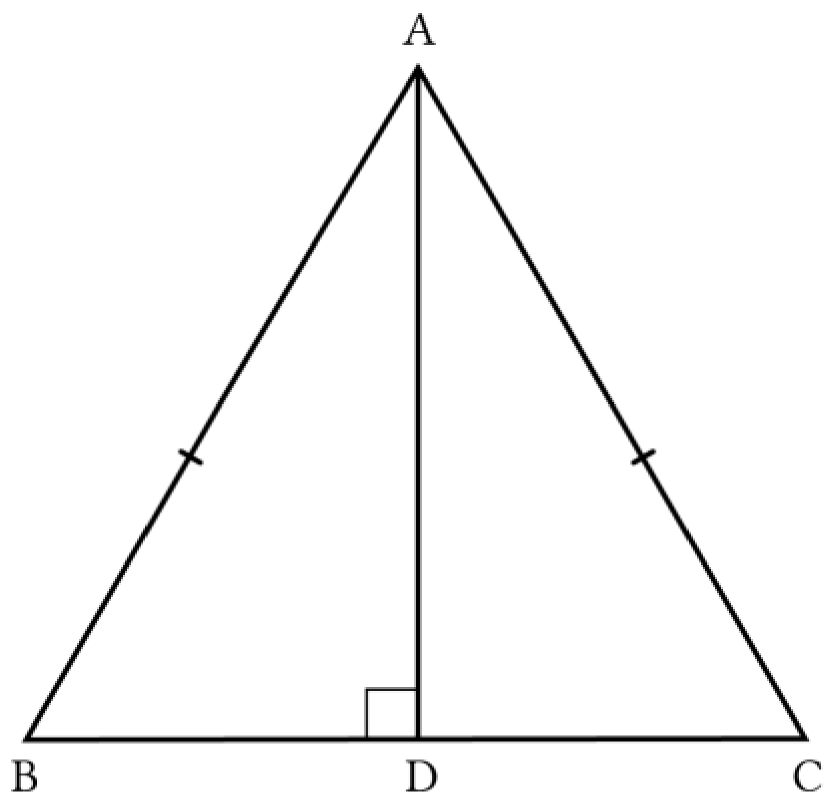
AD is an altitude of an isosceles triangle ABC in which

$AB = AC$. **Show that:**

(i)AD bisects BC

(ii)AD bisects $\angle A$.

Solution:



Given,

AD is an altitude and $AB = AC$

(i) In ΔABD and ΔACD,

$\angle ADB = \angle ADC = 90°$

$AB = AC$ (Given)

$AD = AD$ (Common)

Therefore,

$\Delta ABD \cong \Delta ACD$ by RHS congruence condition.

Now,

$BD = CD$ (By CPCT)

Thus, AD bisects BC

(ii) $\angle BAD = \angle CAD$ (By CPCT)

Thus, AD bisects $\angle A$.

Question: 3

Two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of ΔPQR (See Fig. 7.40). Show that:

(i) $\Delta ABM \cong \Delta PQN$

(ii) $\Delta ABC \cong \Delta PQR$

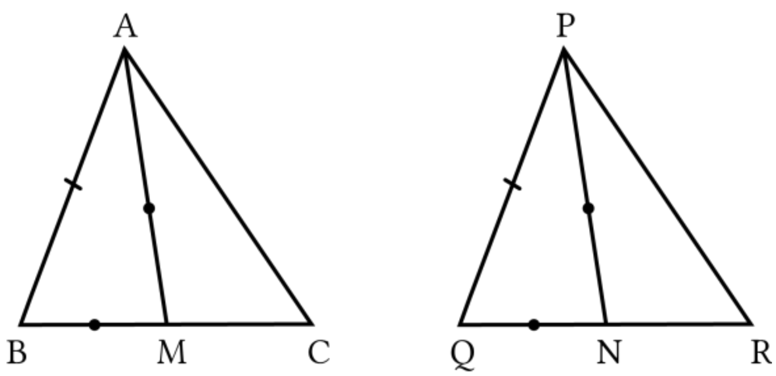


Fig. 7.40

Solution:

Given,

$AB = PQ, BC = QR$ and $AM = PN$

(i) $\frac{1}{2}BC = BM$ and $\frac{1}{2}QR = QN$ (AM and PN are medians)

also,

$BC = QR$ (Given)

$\Rightarrow \frac{1}{2}BC = \frac{1}{2}QR$

$\Rightarrow BM = QN$

In ΔABM and ΔPQN,

$AM = PN$ (Given)

$AB = PQ$ (Given)

$BM = QN$ (As proved above)

Therefore, $\Delta ABM \cong \Delta PQN$ by SSS congruence condition.

(ii)In ΔABC and ΔPQR,

$AB = PQ$ (Given)

$\angle ABC = \angle PQR$ by CPCT,

Since, $\Delta ABM \cong \Delta PQN$

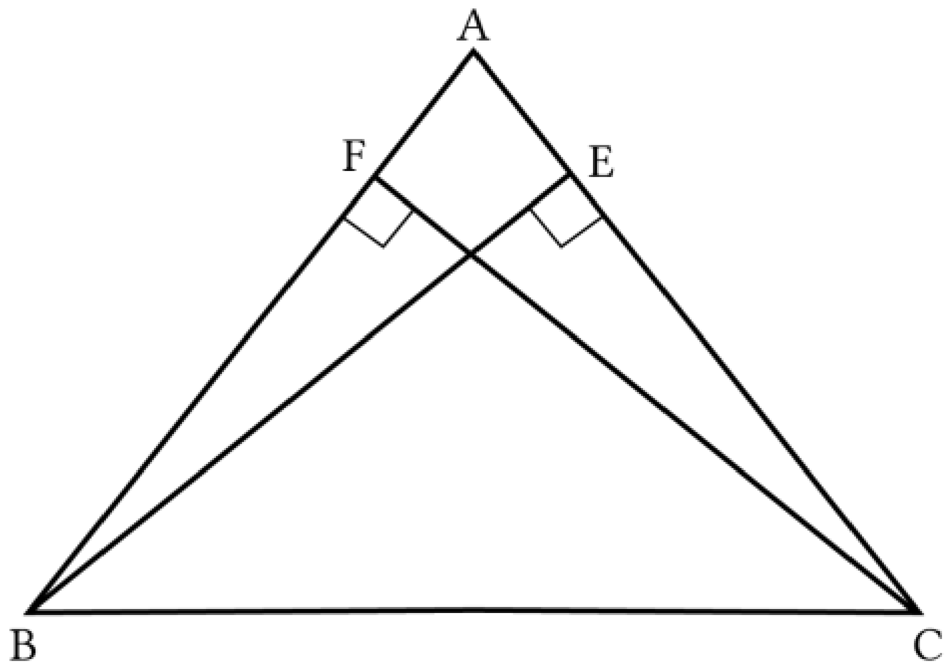
$BC = QR$ (Given)

Therefore, $\Delta ABC \cong \Delta PQR$ by SAS congruence condition.

Question: 4

BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.

Solution:



Given,

$BE = CF$ and $BE \perp AC$ and $CF \perp AB$.

To show triangle ABC is an isosceles triangle.

Now,

In ΔBEC and ΔCFB,

$\angle BEC = \angle CFB = 90°$ (As $BE \perp AC$ and $CF \perp AB$)

$BC = CB$ (Common)

$BE = CF$ (Given)

Therefore, $\Delta BEC \cong \Delta CFB$ by RHS congruence condition.

$\Rightarrow \angle ECB = \angle FBC$ (By CPCT)

$\Rightarrow \angle ACB = \angle ABC$

Thus,

In ΔABC, $AB = AC$

(As sides opposite to the equal angles are equal.)

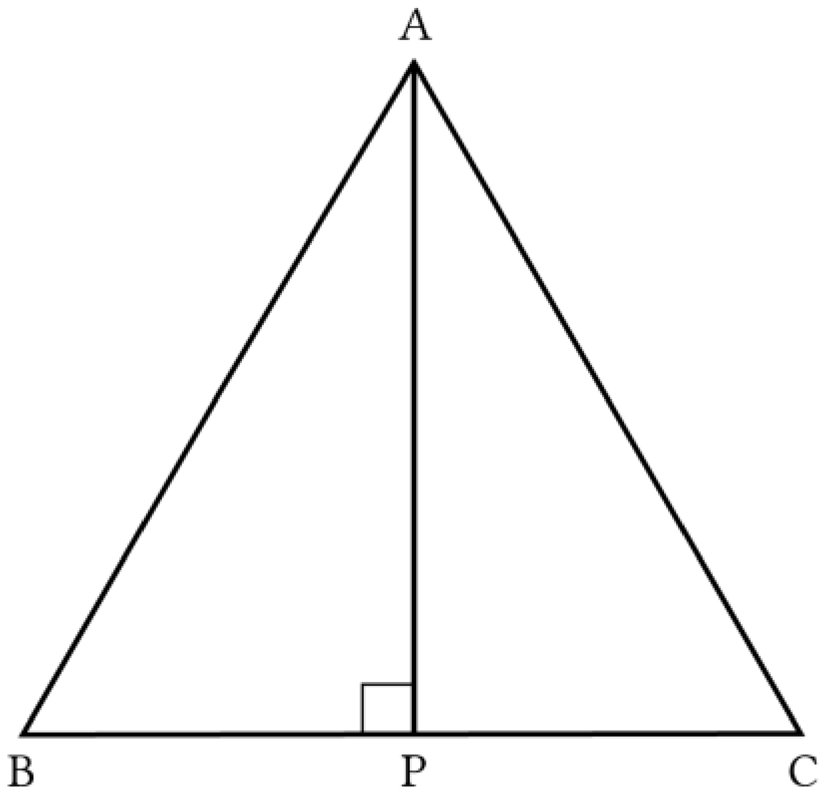
Thus, ΔABC is an isosceles triangle.

Question: 5

ABC is an isosceles triangle with $AB = AC$. Draw $AP \perp BC$.

Show that $\angle B = \angle C$.

Solution:



Given, ABC is an isosceles triangle with

$AB = AC$, and $AP \perp BC$.

In ΔABP and ΔACP,

$\angle APB = \angle APC = 90°$ (Given)

$AB = AC$ (Given)

$AP = AP$ (Common)

Therefore, $\Delta ABP \cong \Delta ACP$ by RHS congruence condition.

Thus,

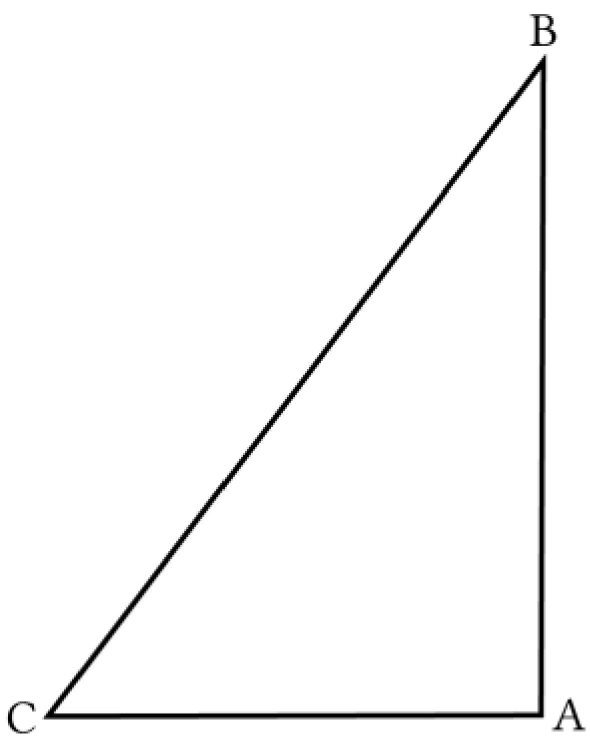
$\angle B = \angle C$ (By CPCT).

Exercise 7.4 (6)

Question: 1

Show that in a right-angled triangle, the hypotenuse is the longest side.

Solution:



Let us consider ΔABC be a triangle right angled at A.

Now,

$\angle A+\angle B+\angle C=180°$ (The sum of all the angles of a triangle is 180°)

$\Rightarrow \angle B + \angle C = 90°$ as $\angle A$ is $90°$.

$\Rightarrow \angle A$ is the largest angle of the triangle, then the side opposite to it must be the longest line segment.

So, BC is the longest side in ΔABC.

Also, BC is the hypotenuse. (Opposite side to the right angle)

Therefore, BC is the hypotenuse which is the longest side of the right-angled triangle ABC.

Question: 2

In Fig. 7.48, sides AB and AC of ΔABC are extended to points P and Q respectively. Also, $\angle PBC < \angle QCB$. Show that $AC > AB$.

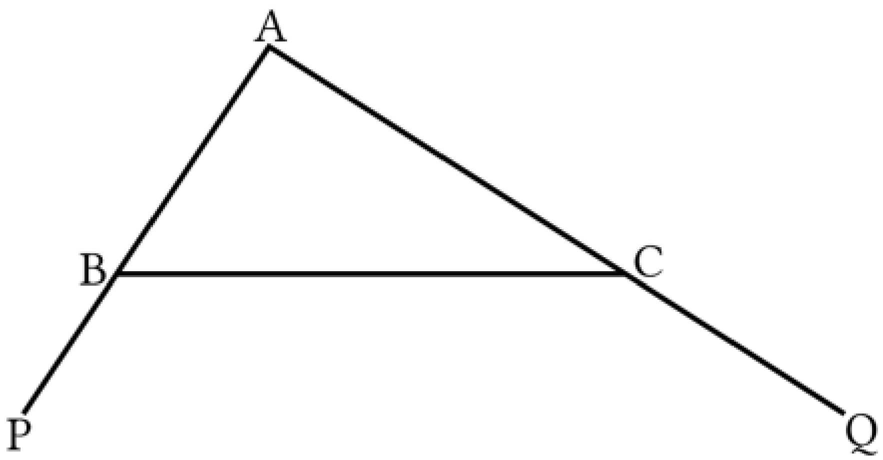


Fig. 7.48

Solution:

Given, sides AB and AC of ΔABC are extended to points P and Q respectively.

Also, $\angle PBC < \angle QCB$

Since, AP is a line segment,

$\Rightarrow \angle ABC + \angle PBC = 180°$

$\Rightarrow \angle ABC = 180° - \angle PBC \quad .....(i)$

Also,

Since, AQ is also a line segment,

$\Rightarrow \angle ACB + \angle QCB = 180°$

$\Rightarrow \angle ACB = 180° - \angle QCB \ ...(ii)$

Now,

$\angle PBC < \angle QCB \ (\text{Given})$

$\Rightarrow 180° - \angle PBC > 180° - \angle QCB$

$\Rightarrow \angle ABC > \angle ACB \left(\text{By using } (i) \ \& \ (ii)\right)$

Thus, $AC > AB$, as side opposite to the greater angle is longer.

Question: 3

In Fig. 7.49, $\angle B < \angle A$ and $\angle C < \angle D$.

Show that $AD < BC$.

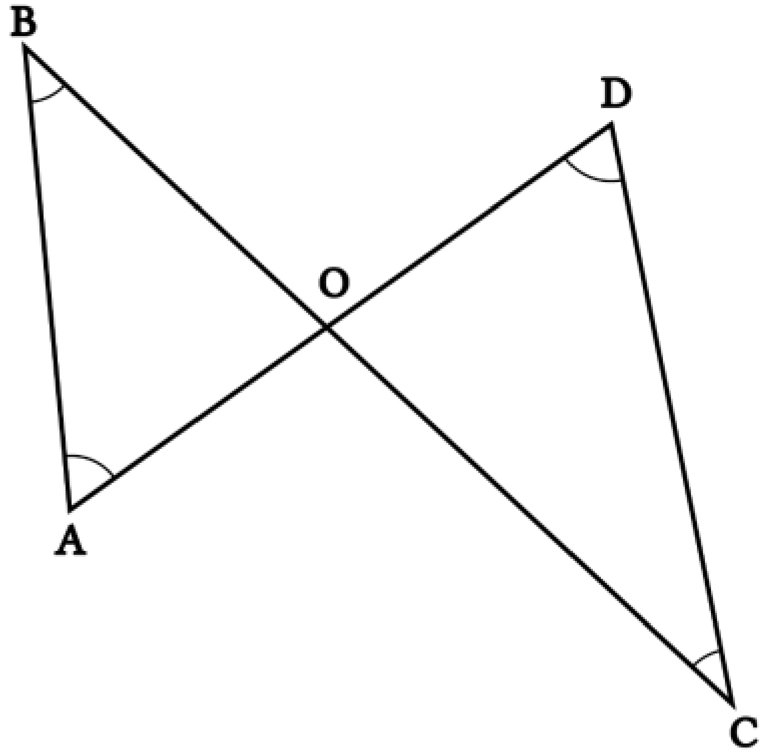


Fig. 7.49

Solution:

Given,

$\angle B < \angle A$ and $\angle C < \angle D$.

In ΔOAB,

Since, $\angle B < \angle A$ (Given)

Therefore, $AO < BO ---(i)$ (Side opposite to the smaller angle is smaller)

Again,

In ΔOCD,

Since $\angle C < \angle D$ (Given)

Therefore, $OD < OC --- (ii)$

(Side opposite to the greater angle is longer)

Adding (i) and (ii),

$$AO + OD < BO + OC$$
$$\Rightarrow AD < BC$$

Hence Proved.

Question: 4

AB and CD are respectively the smallest and longest sides of a quadrilateral ABCD (See Fig. 7.50).

Show that $\angle A > \angle C$ and $\angle B > \angle D$.

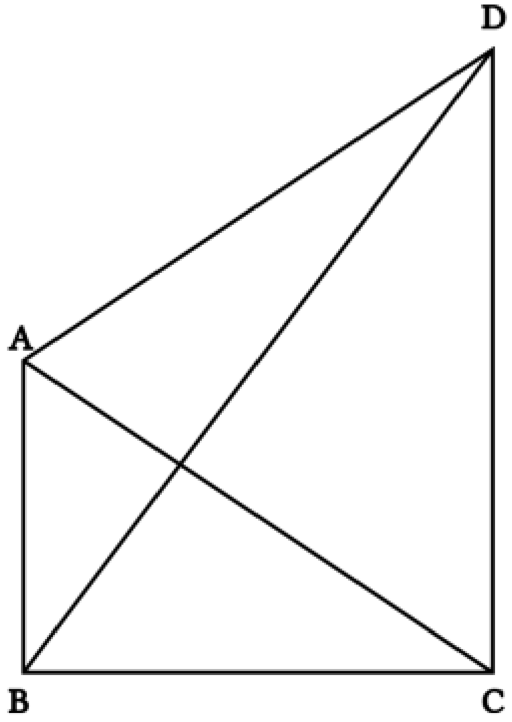


Fig. 7.50

Solution:

Given AB and CD are respectively the smallest and longest sides of a quadrilateral ABCD.

In ΔABD,

Since, AB is the smallest side of quadrilateral ABCD.

$\therefore AB < AD \Rightarrow \angle ADB < \angle ABD ---(i)$

(Angle opposite to the longer side is larger.)

Similarly,

In ΔBCD,

Since, CD is the largest side of quadrilateral ABCD.

$\therefore BC < DC$

$\Rightarrow \angle BDC < \angle CBD ---(ii)$

(Angle opposite to the longer side is larger.)

Adding (i) and (ii) we get,

$\angle ADB + \angle BDC < \angle ABD + \angle CBD$

$\Rightarrow \angle ADC < \angle ABC$

$\Rightarrow \angle D < \angle B$

$\Rightarrow \angle B > \angle D$

Similarly,

In ΔABC,

$AB < BC$ (Since, AB is the smallest side)

$\Rightarrow \angle ACB < \angle BAC ---(iii)$

(Angle opposite to longer side is larger.)

Similarly,

In ΔADC,

$AD < CD$ (Since, CD is the largest side)

$\Rightarrow \angle DCA < \angle DAC ---(iv)$

Adding (iii) and (iv) we get,

$\angle ACB + \angle DCA < \angle BAC + \angle DAC$

$\Rightarrow \angle BCD < \angle BAD$

$\Rightarrow \angle C < \angle A$

$\Rightarrow \angle A > \angle C$

Hence, it is proved.

Question: 5

In Fig 7.51, $PR > PQ$ and PS bisects $\angle QPR$.
Prove that $\angle PSR > \angle PSQ$.

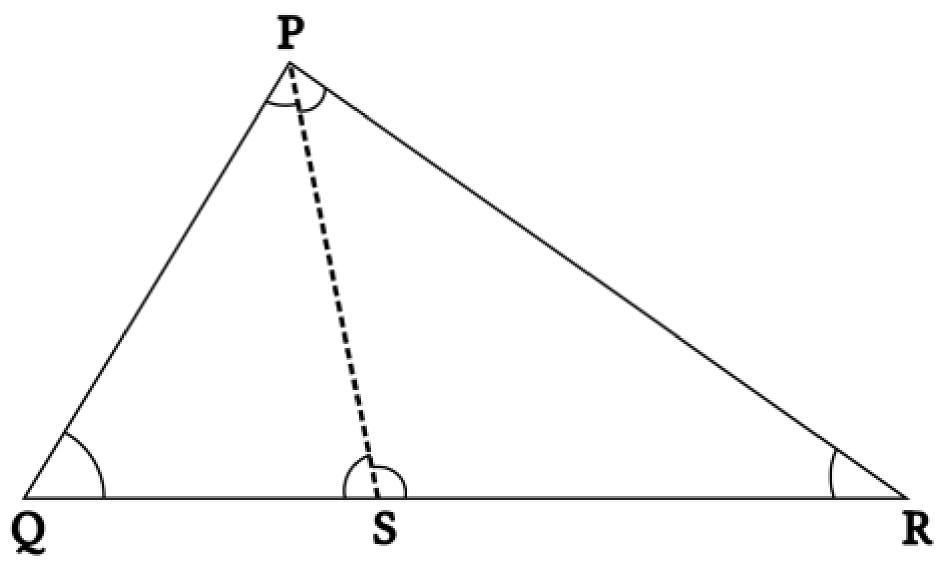


Fig. 7.51

Solution:

Given, PR>PQ and PS bisects $\angle QPR$

In ΔPQR,

PR>PQ (Given)

$\Rightarrow \angle PQR > \angle PRQ ---$(i) ($\because$ Angle opposite to the longer side is larger.)

Now,

$\angle QPS = \angle RPS \ ---(ii)$(PS bisects $\angle QPR$)

In ΔPQS,

$\angle PSR = \angle PQS + \angle QPS \ ---(iii)$(Exterior angle of a triangle is equal to the sum of opposite interior angles.)

Similarly, in ΔPSR,

$\angle PSQ = \angle PRS + \angle RPS \ ---(iv)$(Exterior angle of a triangle is equal to the sum of opposite interior angles)

Adding (i) and (ii) we get,

$\angle PQR + \angle QPS > \angle PRQ + \angle RPS$

Or, $\angle PQS + \angle QPS > \angle PRS + \angle RPS$

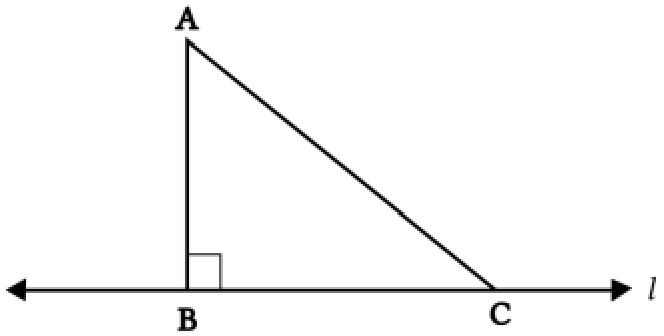
$\Rightarrow \angle PSR > \angle PSQ$ (From (iii) and (iv))

Hence, it is proved.

Question: 6

Show that of all line segments drawn from a given point not on it, the perpendicular line segment is the shortest.

Solution:



Let l be a line segment and B is a point lying on it.

We draw a line AB perpendicular to l. Let C (Different from B) be a point on l.

In ΔABC,

$\angle B = 90°$

Now,

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow \angle A + \angle C = 90° \; (\because \angle B = 90°)$

$\therefore \angle C < \angle B$

$\Rightarrow AB < AC$ (Side opposite to the larger angle is longer.)

Hence, it is proved.

Exercise 7.5 (4)

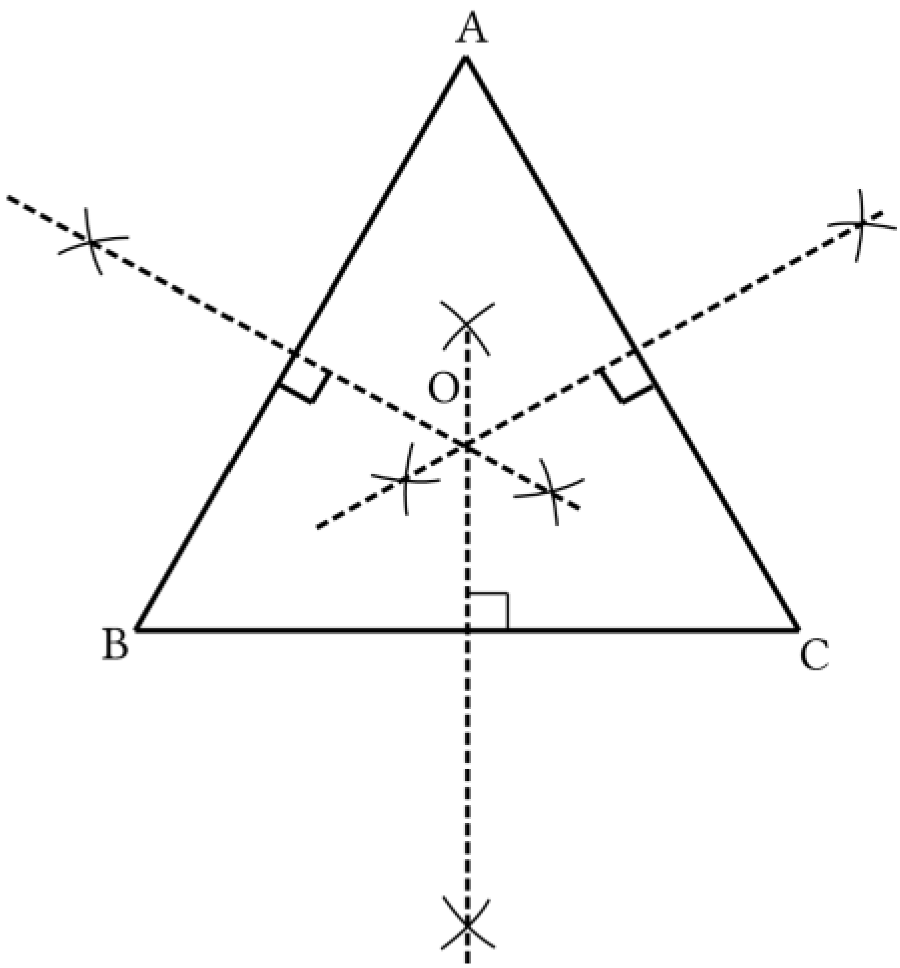
Question: 1

ABC is a triangle. Locate a point in the interior of Δ ABC which is equidistant from all the vertices of Δ ABC.

Solution:

We know that circumcentre of a triangle is always equidistant from all the vertices of that triangle.

So, circumcentre is the point where perpendicular bisectors of all the sides of the triangle meet together.



Thus, in ΔABC, to find the circumcentre, we can draw the perpendicular bisectors of sides AB, BC, and CA of the triangle.

Hence, O is the point where these bisectors are meeting together.

Therefore, O is the point which is equidistant from all the vertices of ΔABC.

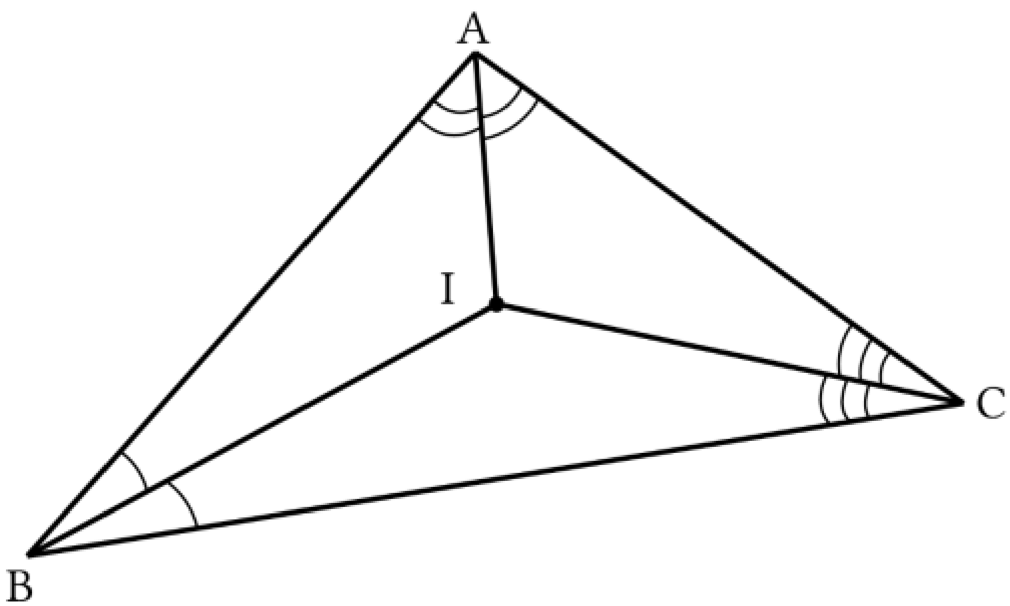
Question: 2

In a triangle locate a point in its interior which is equidistant from all the sides of the triangle.

Solution:

We know that the point which is equidistant from all the sides of a triangle is called the in centre of the triangle.

So, in centre of a triangle is the intersection point of the angle bisectors of the interior angles of the triangle.



Let ABC be a triangle.

Then, in ΔABC, the in-centre of the triangle is the angle bisectors of the interior angles of the triangle.

Let I is the point where these angle bisectors are intersecting each other.

Therefore, I is the point which is equidistant from all the sides of ΔABC.

Question: 3

In a huge park, people are concentrated at three points (see Fig. 7.52):

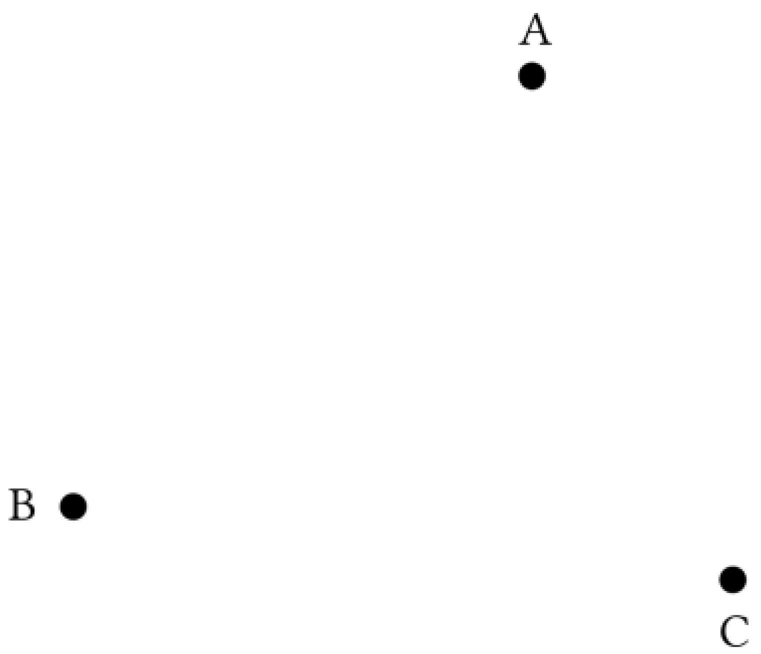


Fig. 7.52

A: where there are different slides and swings for children,

B: near which a man-made lake is situated, C: which is near to a large parking and exit

Where should an ice cream parlour be set up so that maximum number of persons can approach it?

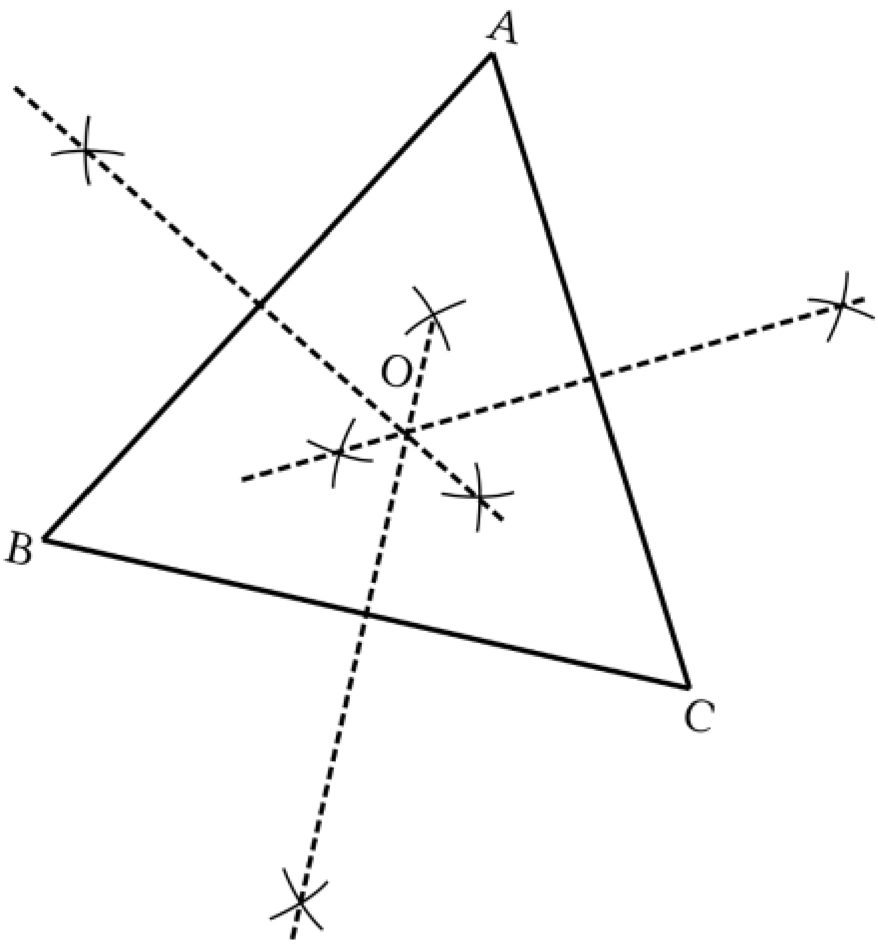
(Hint: The parlour should be equidistant from A, B and C)

Solution:

To find the point where maximum number of persons can approach the ice-cream parlour, we need to find the point which is equidistant from A, B and C.

Now, A, B and C form a triangle. Thus circumcentre is the only point that is equidistant from its vertices A, B and C.

So, the ice-cream parlour should be set up at the circumcentre O of ΔABC.



We know that circumcentre O of the triangle is the intersecting point of perpendicular bisectors of the sides of the triangle.

Question: 4

Complete the hexagonal and star shaped Rangolies [see Fig. 7.53 (i) and (ii)] by filling them with as many equilateral triangles of side 1 cm as you can. Count the number of triangles in each case. Which has more triangles?

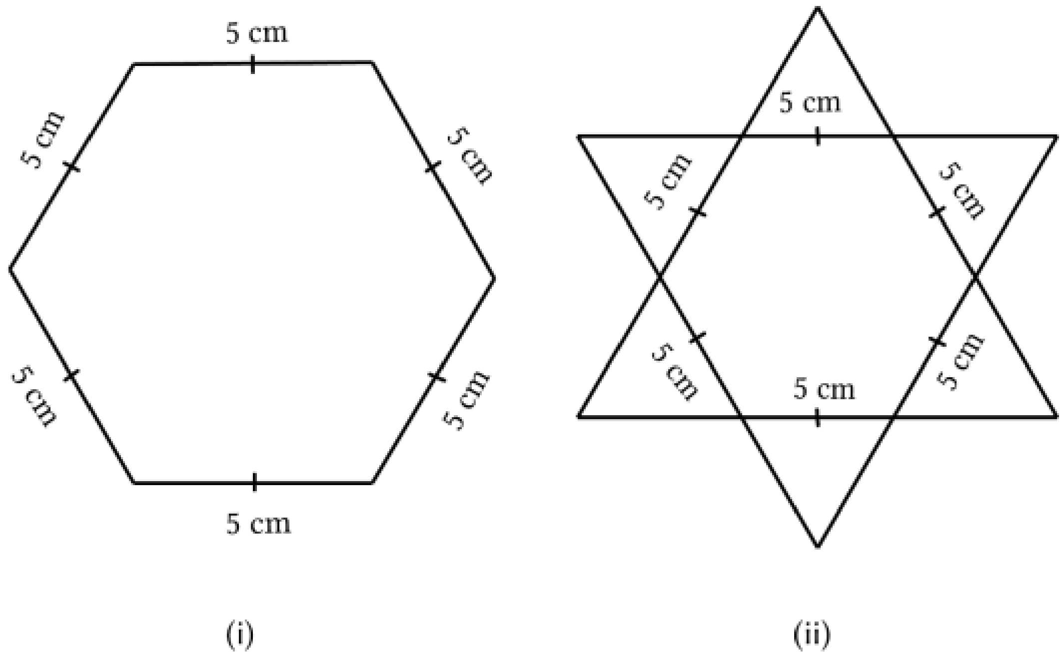
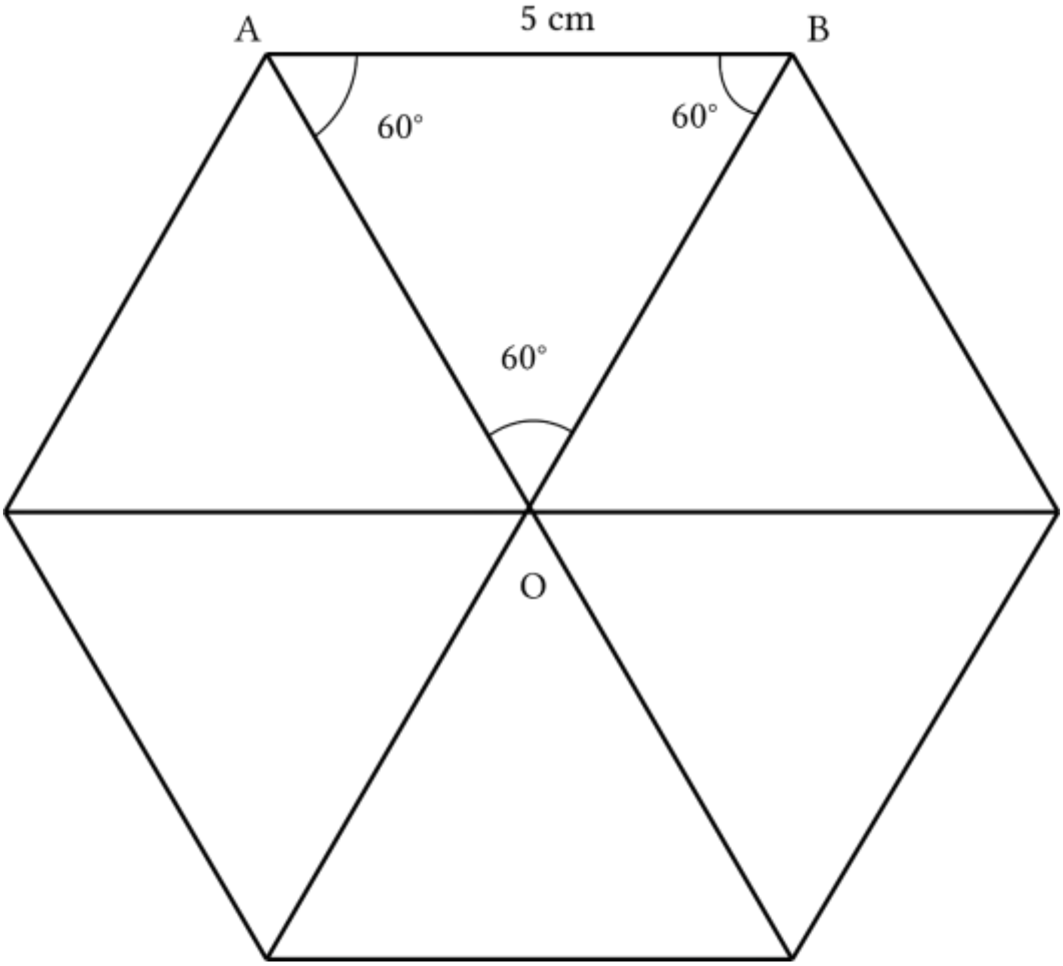


Fig. 7.53

Solution:

We can observe that hexagonal-shaped rangoli has 6 equilateral triangles in it.



$$= \frac{\sqrt{3}}{4}(\text{side}) = \frac{\sqrt{3}}{4}(5)^2$$

Now, area of ΔOAB

$$= \frac{\sqrt{3}}{4}(25) = \frac{25\sqrt{3}}{4}\text{cm}^2$$

Thus, area of hexagonal-shaped rangoli

$$= 6 \times \frac{25\sqrt{3}}{4} = \frac{75\sqrt{3}}{2}\text{cm}^2$$

Now, area of equilateral triangle having its side as

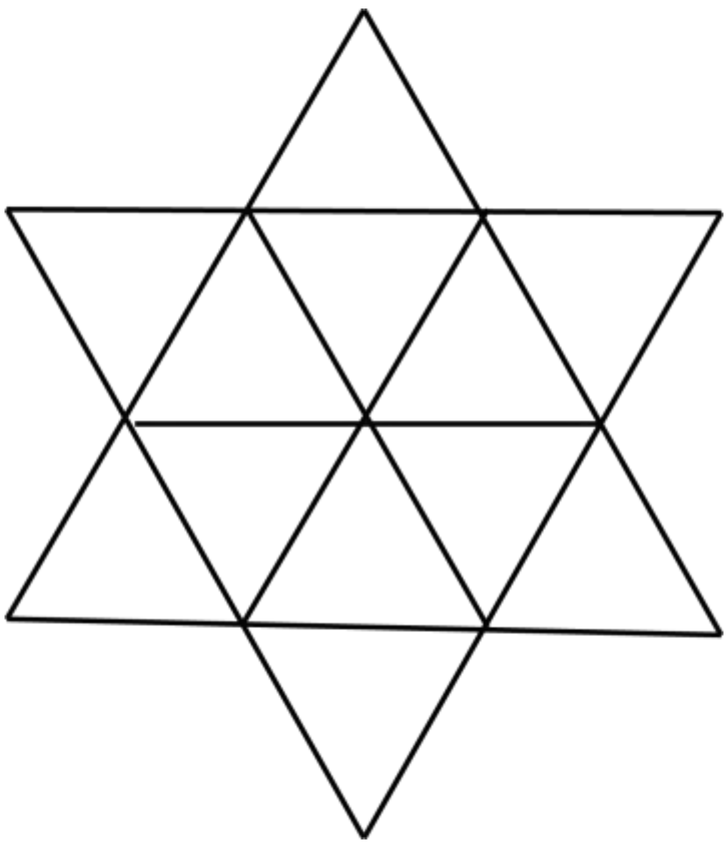
$$1\text{ cm} = \frac{\sqrt{3}}{4}(1)^2 = \frac{\sqrt{3}}{4}\text{ cm}^2$$

So, number of equilateral triangles of 1 cm side that can

be filled in this hexagonal-shaped rangoli $= \dfrac{\frac{75\sqrt{3}}{2}}{\frac{\sqrt{3}}{4}} = 150$

Therefore, star-shaped rangoli has 12 equilateral triangles

of side 5cm in it.



Here, area of star-shaped rangoli $= 12 \times \frac{\sqrt{3}}{4} \times (5)^2 = 75\sqrt{3}$

So, number of equilateral triangles of 1cm side that can be filled in star-shaped rangoli $= \dfrac{75\sqrt{3}}{\dfrac{\sqrt{3}}{4}} = 300$

Therefore, star-shaped rangoli has more equilateral triangles in it.