NCERT Exemplar

Class 9 Maths

Chapter 6- Lines and Angles

Exercise 6.1 (8 Multiple Choice Questions and Answers)

Question: 1

In Fig. 6.1, if $AB \parallel CD \parallel EF,\ PQ \parallel RS, \angle RQD = 25^\circ$ **and** $CQP = 60^\circ$, **then** $\angle QRS$ **is equal to:**

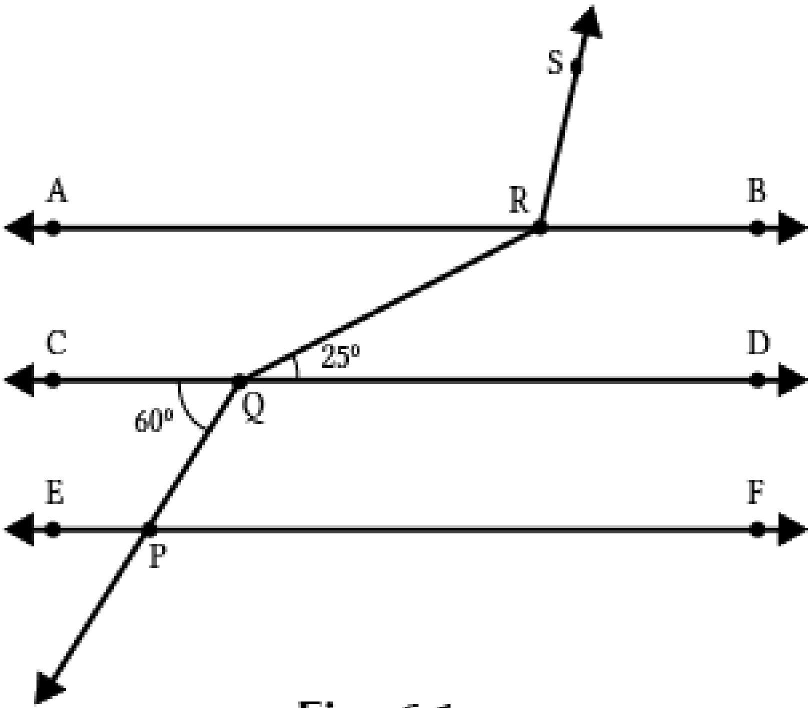


Fig. 6.1

(a) 85°

(b) 135°

(c) 145°

(d) 110°

Solution:

c

Given that,

$PQ \parallel RS$

$\therefore \angle PQC = \angle BRS = 60^\circ$

[Alternate exterior angles and]

and $\angle DQR = \angle QRA = 25^\circ$ [Alternate interior angles and

$\angle DQR = 25^\circ$]

$\therefore \angle QRS = \angle QRA + \angle ARS = \angle QRA + (180^\circ - \angle BRS) \angle PQC = 60^\circ$

[Linear pair]

$= 25^\circ + 180^\circ - 60^\circ$

$= 205^\circ - 60^\circ$

$= 145^\circ$

Hence, option (c) is correct.

Question: 2

If one angle of a triangle is equal to the sum of the other two angles, then the triangle is:

(a) an isosceles triangle

(b) an obtuse triangle

(c) an equilateral triangle

(d) a right triangle

Solution:

d

Question: 3

An exterior angle of a triangle is and its two interior opposite angles are equal. Each of these equal angles is:

(a) $37\frac{1}{2}^{\circ}\ 105^{\circ}$

(b) $52\frac{1}{2}^{\circ}$

(c) $72\frac{1}{2}^{\circ}$

(d) 75°

Solution:

b

Given,

Exterior angle = 105°

Let one of interior angle be x°.

$\because$ Sum of two opposite interior angles $=$ Exterior angle

$\therefore\ x^\circ + x^\circ = 105^\circ$

$\Rightarrow 2x^\circ = 105^\circ$

$\therefore\ x^\circ = \dfrac{105^\circ}{2}$

$x^\circ = 52\dfrac{1}{2}^\circ$

Hence, each equal angle of triangle is $52\dfrac{1}{2}^\circ$

Question 4

The angles of a triangle are in the ratio 5 : 3 : 7. The triangle is:

(a) an acute angled triangle

(b) an obtuse angled triangle

(c) a right triangle

(d) an isosceles triangle

Solution:

a

Given that,

The ratio of angles of a triangle is 5 : 3 : 7.

Let the angles of a triangle be $\angle A$, $\angle B$ and $\angle C$.

Let, $\angle A = 5x, \angle B = 3x$ and $\angle C = 7x$

In ΔABC, $\angle A + \angle B + \angle C = 180^\circ$

(The sum of the angles of a triangle is 180°.)

$\therefore\ 5x + 3x + 7x = 180$

$\Rightarrow 15x = 180^\circ$

$\Rightarrow x = 12^\circ$

$\therefore\ \angle A = 5x = 5 \times 12^\circ = 60^\circ$

$\angle B = 3x = 3 \times 12^\circ = 36^\circ$

and $\angle C = 7x = 7 \times 12^\circ = 84^\circ$

Since all the angles are less than 90°, the triangle is an acute angled triangle.

Question: 5

If one of the angles of a triangle is 130°, then the angle formed when the bisectors of the other two angles meet can be:

(a) 50°

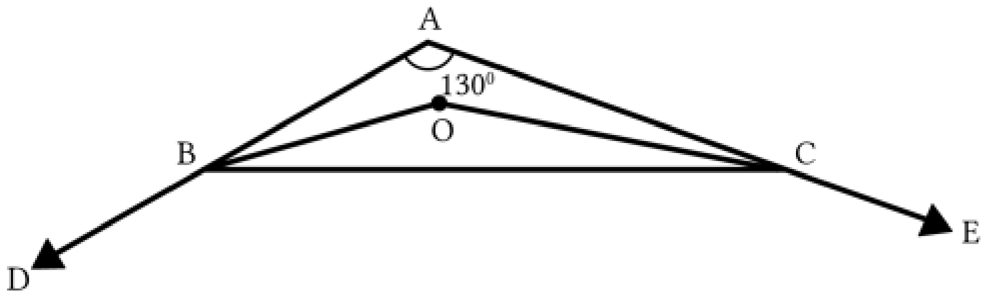
(b) 65°

(c) 145°

(d) 155°

Solution:

d



Let the angles of ΔABC be $\angle A, \angle B$ and $\angle C$.

In ΔABC,

$\angle A + \angle B + \angle C = 180^{\circ}$

(The sum of the angles of a triangle is 180°.)

$$\Rightarrow \frac{1}{2}\angle A + \frac{1}{2}\angle B + \frac{1}{2}\angle C = \frac{180^\circ}{2}$$

(Dividing both the sides by 2)

$$\Rightarrow \frac{1}{2}\angle B + \frac{1}{2}\angle C = \frac{180^\circ}{2} - \frac{1}{2}\angle A$$

$$\Rightarrow \frac{1}{2}\angle B + \frac{1}{2}\angle C = 90^\circ - \frac{1}{2}\angle A \Rightarrow \frac{1}{2}\angle B + \frac{1}{2}\angle C = 90^\circ - \frac{1}{2}\angle A$$

Now, OB and OC are the angle bisectors of

$\angle ABC$ and $\angle ACB$

Hence, $\angle OBC = \frac{1}{2}\angle B$.

And $\angle BCO = \frac{1}{2}\angle C$.

In ΔOBC, $\angle OBC + \angle BCO + \angle COB = 180^\circ$

$$\Rightarrow \frac{1}{2}\angle B + \frac{1}{2}\angle C = 180^\circ - \angle COB$$

$\Rightarrow 90^\circ - \frac{1}{2}\angle A = 180^\circ - \angle COB$ (From equation i)

$$\Rightarrow \angle COB = 90^\circ + \frac{1}{2}\angle A$$

$$\Rightarrow \angle COB = 90^\circ + \frac{130^\circ}{2}$$

$$\Rightarrow \angle COB = 90^\circ + 65^\circ$$

$$\Rightarrow \angle COB = 155^\circ$$

Question: 6

In Fig. 6.2, POQ is a line. The value of *x* is:

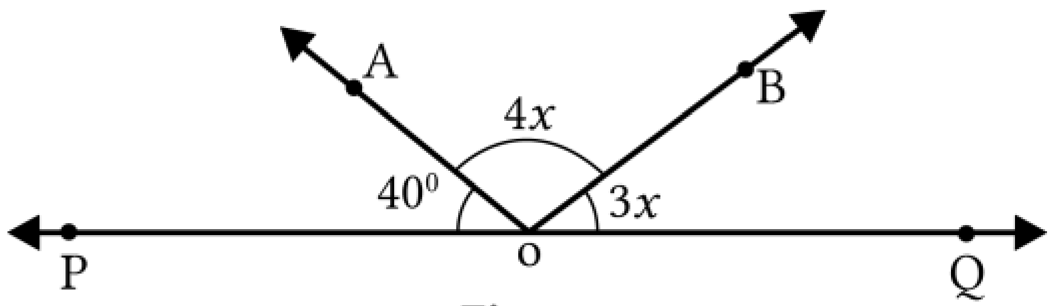


Fig. 6.2

(a) 20°

(b) 25°

(c) 30°

(d) 35°

Solution:

a

Given,

POQ is a line segment.

Hence, $\angle POQ = 180^\circ$

$\Rightarrow \angle POA + \angle AOB + \angle BOQ = 180^\circ$

Putting $\angle POA = 40^\circ$, $\angle AOB = 4x$, and $\angle BOQ = 3x$

$\Rightarrow 40^\circ + 4x + 3x = 180^\circ$

$\Rightarrow \mathbf{7x = 140^\circ}$

$\Rightarrow \mathbf{7x = 140^\circ}$

$\Rightarrow \mathbf{x = 20^\circ}$

Question 7

In Fig. 6.3, if $OP \parallel RS$, $\angle OPQ = 110^\circ$ and $\angle QRS = 130^\circ$, then $\angle PQR$ is equal to:

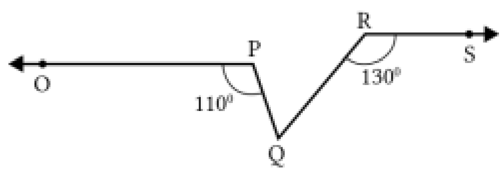


Fig. 6.3

(a) 40°

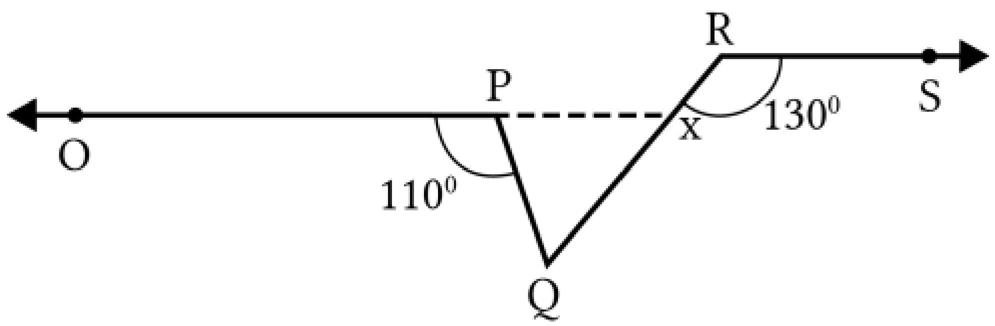
(b) 50°

(c) 60°

(d) 70°

Solution:

c



producing OP such that it intersects RQ at point *x*.

Now, $OP \parallel RS$ and RX is transversal.

Hence, $\angle RXP = \angle QRS$

[The alternate interior angles are equal.]

$\Rightarrow \angle RXP = 130^\circ$ ... (i) $[\angle QRS = 130^\circ]$

Now, RQ is a line segment.

So, $\angle PXQ + \angle RXP = 180^\circ$

$\Rightarrow \angle PXQ = 180^\circ - \angle RXP$

$\Rightarrow \angle PXQ = 180^\circ - 130^\circ$ [From eqn (i)]

$\Rightarrow \angle PXQ = 50^\circ$

In ΔPQX, $\angle OPQ$ is an exterior angle.

Hence, $\angle OPQ = \angle PXQ + \angle PQX$

[∴ Exterior angle = sum of two opposite interior angles.]

$\Rightarrow 110^\circ = 50^\circ + \angle PQX$

$\Rightarrow \angle PQX = 110^\circ - 50^\circ$

$\Rightarrow \angle PQX = 60^\circ$

Question 8

Angles of a triangle are in the ratio 2 : 4 : 3.

The smallest angle of the triangle is

(a) 60°

(b) 40°

(c) 80°

(d) 20°

Solution:

b

Given that,

The ratio of angles of a triangle is 2 : 4 : 3.

Let angles of a triangle be $\angle A$, $\angle B$ and $\angle C$.

$\therefore \angle A = 2x, \angle B = 4x$ and $\angle C = 3x$

In ΔABC, $\angle A + \angle B + \angle C = 180^\circ$

[The sum of the angles of a triangle is 180°]

$\therefore \quad 2x + 4x + 3x = 180$

$\Rightarrow \mathbf{9}x = \mathbf{180}^\circ$

$\Rightarrow x = 20^\circ$

$\therefore \angle \mathbf{A} = \mathbf{2}x = \mathbf{2} \times \mathbf{20}^\circ = \mathbf{40}^\circ$

$\angle B = 4x = 4 \times 20^\circ = 80^\circ$

and $\angle C = 3x = 3 \times 20^\circ = 60^\circ$

Hence, option (b) is correct.

Exercise 6.2

Question 1

For what value of $x + y$ in Fig. 6.4 will ABC be a line? Justify your answer.

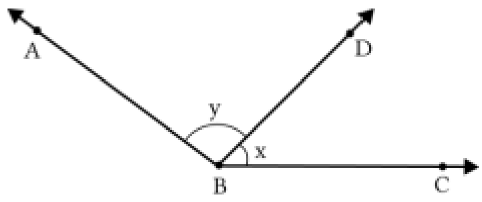


Fig. 6.4

Solution:

For ABC to be a line, the sum of the two adjacent angles must be 180° or $\angle ABD$ and $\angle DBC$ must form a linear pair.

i.e., $x + y = 180^\circ$.

Question 2

Can a triangle have all the angles less than 60°? Give reason for your answer.

Solution:

No, a triangle cannot have all angles less than 60°, because if all angles are less than 60°, then their sum will be less than 180° (not equal to 180°).

Hence, it will not be a triangle.

Question: 3

Can a triangle have two obtuse angles? Give reason for your answer.

Solution:

No, because if a triangle has two obtuse angles i.e., more than 90° angle, then the sum of all three angles of a triangle will be greater than 180°

(not equal to 180°).

Hence, it will not be a triangle.

Question: 4

How many triangles can be drawn with angles measuring as 45°, 64° and 72°? Give reason for your answer.

Solution:

None.

The sum of given angles $= 45^\circ + 64^\circ + 72^\circ = 181^\circ \neq 180^\circ$.

Hence, we see that sum of all three angles is not equal to 180°. So, we cannot draw a triangle with the given angles.

Question: 5

How many triangles can be drawn with angles as 53°, 64° and 63°? Give reason for your answer.

Solution:

The sum of the given angles $= 53^\circ + 64^\circ + 63^\circ = 180^\circ$.

Since, the sum of all interior angles of the triangle is 180°, infinitely many triangles can be drawn.

Question: 6

In Fig. 6.5, find the value of *x* for which the lines *l* and *m* are parallel.

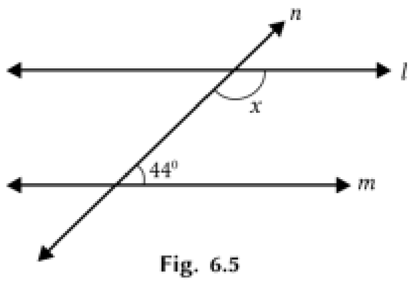


Fig. 6.5

Solution:

In the given figure, $l \parallel m$.

Using the properties of parallel line (if a transversal intersects two parallel lines, then the sum of interior angles on the same side of a transversal is supplementary), we have:

$x + 44^\circ = 180^\circ$

$\Rightarrow x = 180^\circ - 44^\circ$

$\Rightarrow x = 136^\circ$

Question: 7

Two adjacent angles are equal. Is it necessary that each of these angles will be a right angle? Justify your answer.

Solution:

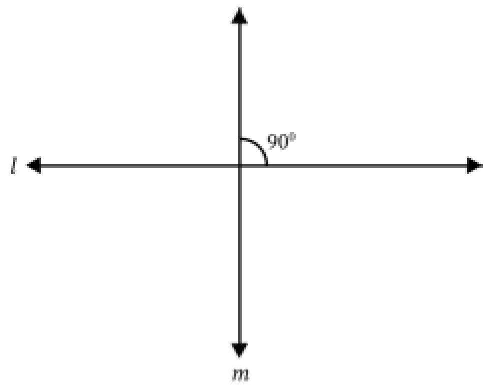
No, because each of these will be a right angle only when they form a linear pair.

Question: 8

If one of the angles formed by two intersecting lines is a right angle, what can you say about the other three angles? Give reason for your answer.

Solution:

Let two intersecting lines be l and m. As one of the angles is a right angle, then it means that lines l and m are perpendicular to each other. By using linear pair axiom, other three angles will be right angles.



Question: 9

In Fig. 6.6, which of the two lines are parallel and why?

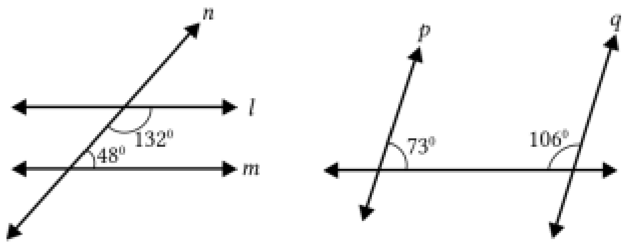


Fig. 6.6

Solution:

In case (i),

The sum of two interior angles $132^\circ + 48^\circ = 180^\circ$.

Here, the sum of two interior angles on the same side of line *n* is 180°. Hence, *l* and *m* are the parallel lines.

In case (ii)

The sum of two interior angles $73^\circ + 106^\circ = 179^\circ \neq 180^\circ$.

Here, the sum of two interior angles on the same side of the transversal is not equal to 180°. Hence, lines *p* and *q* are not the parallel lines.

Question: 10

Two lines *l* and *m* are perpendicular to the same line *n*. Are *l* and *m* perpendicular to each other? Give reason for your answer.

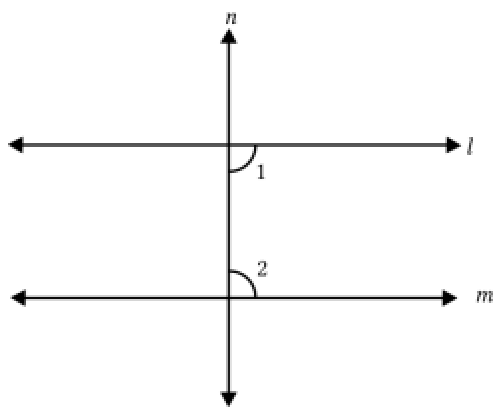
Solution:

No.

Let lines *l* and *m* be two lines which are perpendicular to the line *n*.

$\therefore \angle 1 + \angle 2 = 90^{\circ} + 90^{\circ} = 180^{\circ}$

$[\because l \perp n$ and $m \perp n]$



Exercise: 6.3

Question: 1

In Fig. 6.9, OD is the bisector of $\angle AOC$, *OE* is the bisector of $\angle BOC$ and $OD \perp OE$. Show that the points A, O and B are collinear.

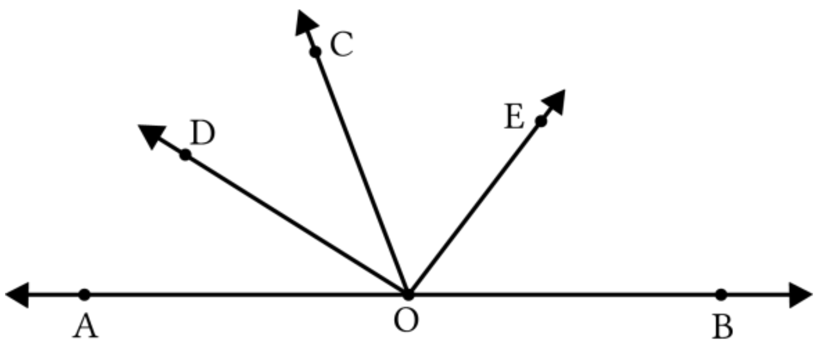


Fig. 6.9

Solution:

Given:

In figure, $OD \perp OE$ and OD is the bisector of $\angle AOC$ and OE is the bisector of $\angle BOC$.

To prove:

Points A, O and *B* are collinear i.e., AOB is a straight line.

Proof:

OD and OE bisect angles $\angle AOC$ and $\angle BOC$ respectively.

$\therefore \angle AOC = 2\angle DOC$ .... (i) and

$\angle BOC = 2\angle COE$ .... (ii)

On adding equations (i) and (ii), we get

$\angle AOC + \angle COB = 2\angle DOC + 2\angle COE$

$\Rightarrow \angle AOC + \angle COB = 2(\angle DOC + \angle COE)$

$\Rightarrow \angle AOC + \angle COB = 2\angle DOE$

$\Rightarrow \angle AOC + \angle COB = 2 \times 90^\circ [\because OD \perp OE]$

$\Rightarrow \angle AOC + \angle COB = 180^\circ$

$\therefore \angle AOB = 180^\circ$

So, $\angle AOC$ and $\angle COB$ are linear pairs or AOB is a straight line.

Hence, points A, O and B are collinear.

Question: 2

In Fig. 6.10, $\angle 1 = 60^\circ$ and $\angle 6 = 120^\circ$. Show that the lines *m* and *n* are parallel.

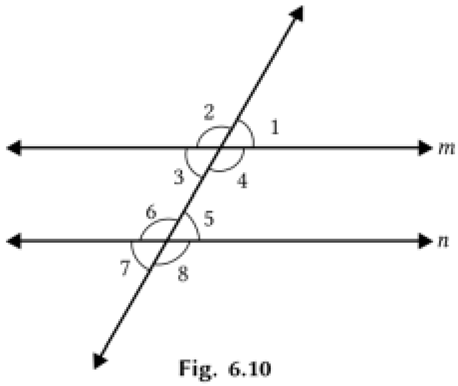


Fig. 6.10

Solution:

Given:

In figure, $\angle 1 = 60^\circ$ and $\angle 6 = 120^\circ$

To prove:

$m \parallel n$.

Proof:

Since, $\angle 1 = 60^\circ$ and $\angle 6 = 120^\circ$

Here, $\angle 1 = \angle 3$ [Vertically opposite angles]

$\therefore \angle 3 = \angle 1 = 60^\circ$

Now, $\therefore \angle 3 + \angle 6 = 60^\circ + 120^\circ$

$\Rightarrow \angle 3 + \angle 6 = 180^\circ$

The sum of two interior angles on the same side of the transversal is 180°.

Hence, the lines are parallel or $m \parallel n$.

Question: 3

AP and BQ are the bisectors of the two alternate interior angles formed by the intersection of a transversal *t* with parallel lines *l* and *m* (Fig. 6.11). Show that $AP \parallel BQ$.

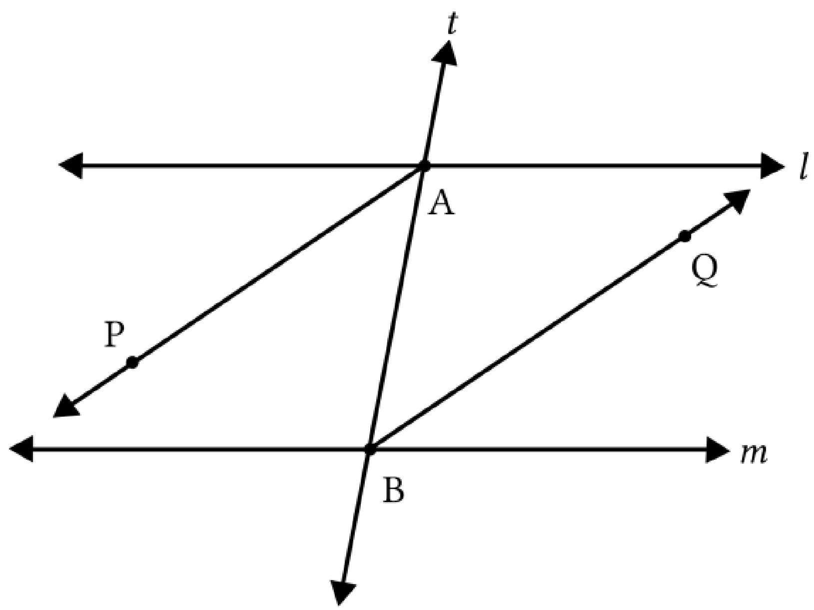


Fig. 6.11

Solution:

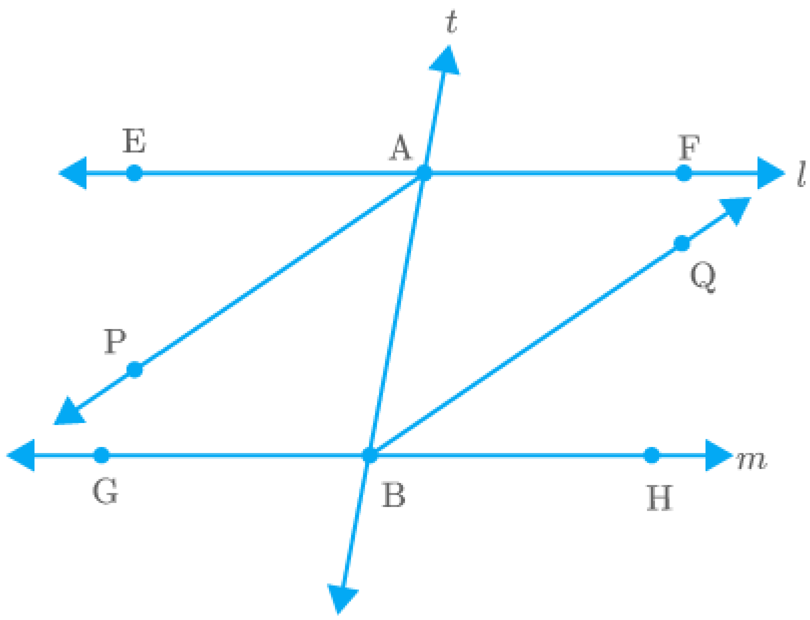


Fig. 6.11

Given:

In figure, $l \parallel m$, AP and BQ are the bisectors of

$\angle EAB$ and $\angle ABH$ respectively.

To prove:

$AP \parallel BQ$

Proof:

Since, $l \parallel m$ and *t* is transversal, therefore, $\angle EAB = \angle ABH$ [Alternate interior angles]

$$\Rightarrow \frac{1}{2}\angle EAB = \frac{1}{2}\angle ABH$$ [Dividing both sides by 2]

$$\Rightarrow \angle PAB = \angle ABQ$$

[AP and BQ are the bisectors of $\angle EAB$ and $\angle ABH$]

Since, $\angle PAB$ and $\angle ABQ$ are alternate interior angles formed when transversal AB intersects lines AP and BQ, hence, $AP \parallel BQ$.

Question: 4

If in Fig. 6.11, bisectors AP and BQ of the alternate interior angles are parallel, then show that $l \parallel m$.

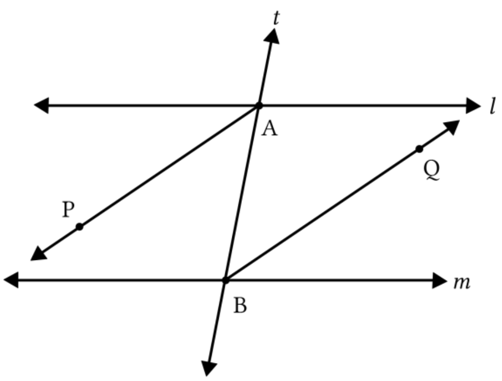


Fig. 6.11

Solution:

Given:

In figure, $AP \parallel BQ$, AP and BQ are the bisectors of alternate interior angles $\angle EAB$ and $\angle ABH$.

To prove:

$l \parallel m$

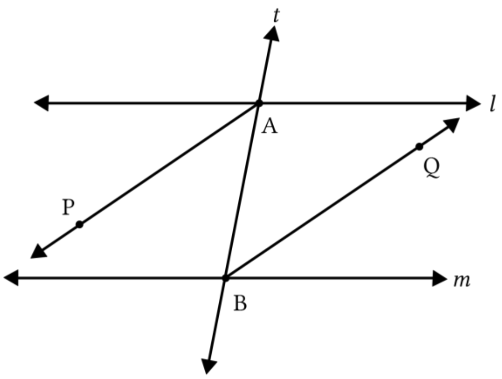


Fig. 6.11

Proof:

Since, $AP \parallel BQ$ and t is the transversal, therefore

$\angle PAB = \angle ABQ$ [Alternate interior angles].

$\Rightarrow 2\angle \mathbf{PAB} = \mathbf{2}\angle \mathbf{ABQ}$ [Multiplying both sides by 2]

$\Rightarrow \angle EAB = \angle ABH$

As the alternate interior angles $\angle EAB$ and $\angle ABH$ are equal, the lines *l* and *m* are parallel. [If two alternate interior angles are equal, then the lines are parallel.]

Question: 5

In Fig. 6.12, $BA \parallel ED$ and $BC \parallel EF$. Show that

$\angle ABC = \angle DEF$

[Hint: Produce DE to intersect BC at P (say)].

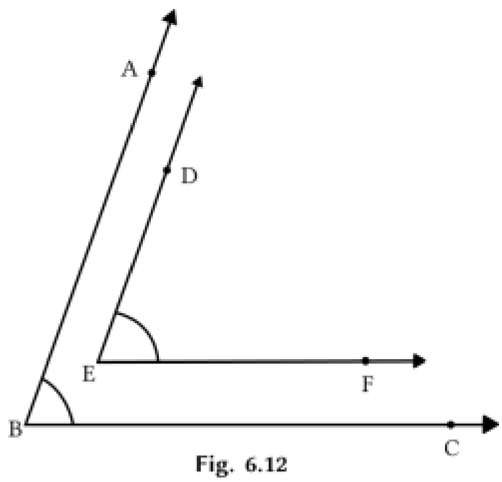


Fig. 6.12

Solution:

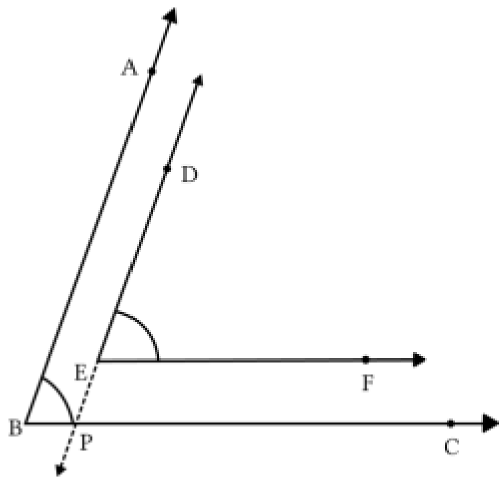
Given:

$BA \parallel ED$, and $BC \parallel EF$.

To prove: $\angle ABC = \angle DEF$

Construction:

Produce DE which intersects BC at P.



Proof:

In figure,

$BA \parallel ED \Rightarrow BA \parallel DP$

$\therefore \angle ABP = \angle EPC$ [Corresponding angles]

$\Rightarrow \angle ABC = \angle EPC$ ... (i)

Again, $BC \parallel EF \Rightarrow PC \parallel EF$

$\therefore \angle EPC = \angle DEF$ ... (ii)

[Corresponding angles]

From equations (i) and (ii), we get

$\therefore \angle ABC = \angle DEF$

Hence, it is proved.

Question: 6

In Fig. 6.13, $BA \parallel ED$ and $BC \parallel EF$.

Show that $\angle ABC + \angle DEF = 180^\circ$.

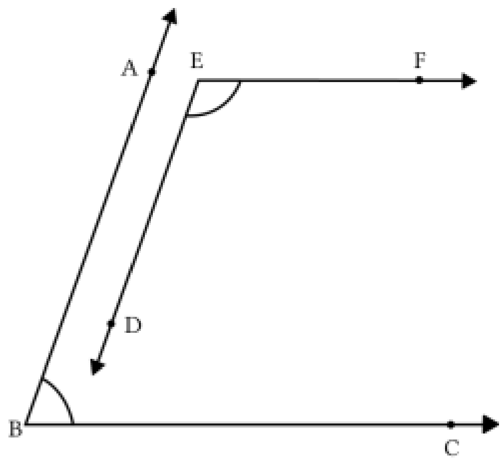


Fig. 6.13

Solution:

Given:

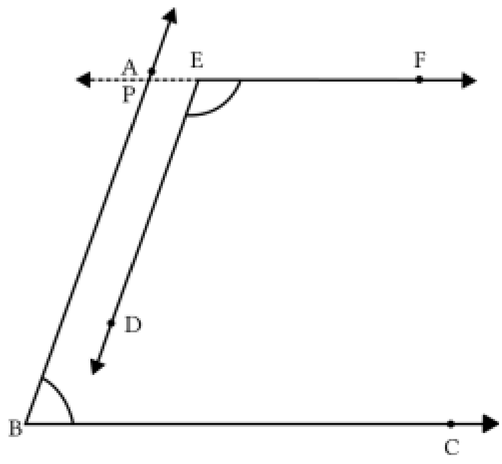
In the figure, $BA \parallel ED$ and $BC \parallel EF$.

To prove:

$\angle ABC + \angle DEF = 180^\circ$.

Construction:

Draw a ray PE opposite to ray EF.



Proof:

In figure, $BC \parallel EF$ or $BC \parallel PF$

$\therefore \angle EPB + \angle PBC = 180^\circ$ ... (i)

[The sum of the interior angles on the same side of the transversal is 180°.]

Now, $AB \parallel ED$ and PE is a transversal line.

$\therefore \angle EPB = \angle DEF$ ... (ii) [Corresponding angles]

From equations (i) and (ii), we get

$\therefore \angle DEF + \angle PBC = 180^\circ$

$\Rightarrow \angle ABC + \angle DEF = 180^\circ$ $[\because \angle PBC = \angle ABC]$

Hence, it is proved.

Question: 7

In Fig. 6.14, $DE \parallel QR$ and AP and BP are the bisectors of $\angle EAB$ and $\angle RBA$, respectively. Find $\angle APB$.

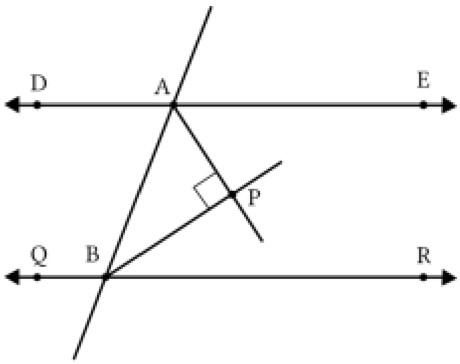


Fig. 6.14

Solution:

Given:

$DE \parallel QR$, and AP and PB are the bisectors of $\angle EAB$ and $\angle RBA$, respectively.

To find:

$\angle APB$

Proof:

As $DE \parallel QR$ and AB is a transversal, the sum of the interior angles on the same side of the transversal is supplementary.

$\therefore \angle EAB + \angle RBA = 180^\circ$

$$\Rightarrow \frac{1}{2}\angle EAB + \frac{1}{2}\angle RBA = \frac{180^\circ}{2}$$

[Dividing both sides by 2]

$$\Rightarrow \frac{1}{2}\angle EAB + \frac{1}{2}\angle RBA = 90^\circ \quad \text{... (i)}$$

Since AP and BP are the bisectors of $\angle EAB$and $\angle RBA$ respectively,

$$\therefore \angle BAP = \frac{1}{2}\angle EAB \quad \text{... (ii)}$$

and $\therefore \angle ABP = \frac{1}{2}\angle RBA$ ... (iii)

On adding equations (ii) and (iii), we get

$$\angle BAP + \angle ABP = \frac{1}{2}\angle EAB + \frac{1}{2}\angle RBA$$

Using equation (i), we get

$$\angle BAP + \angle ABP = 90^\circ \quad \text{... (iv)}$$

In ΔABP,

$\angle BAP + \angle ABP + \angle APB = 180^\circ$

[The sum of the interior angles in a triangle is 180°.]

$\Rightarrow 90^\circ + \angle APB = 180^\circ$

[From equation (iv)]

$\Rightarrow \angle APB = 180^\circ - 90^\circ = 90^\circ$

Hence, $\angle APB = 90^\circ$

Question: 8

The angles of a triangle are in the ratio 2 : 3 : 4. Find the angles of the triangle.

Solution:

Given,

The ratio of the angles of a triangle is 2 : 3 : 4.

Let the angles of a triangle be $\angle A$, $\angle B$ and $\angle C$.

$\therefore \angle A = 2x$, $\angle B = 3x$ and $\angle C = 4x$

In ΔABC, $\angle A + \angle B + \angle C = 180^\circ$

[The sum of the angles of a triangle is 180°]

$\therefore\ 2x+3x+4x+=180^\circ$

$\Rightarrow 9x=180^\circ$

$\Rightarrow x=20^\circ$

$\therefore\ \angle A=2x=2\times 20^\circ=40^\circ$

$\angle B=3x=3\times 20^\circ=60^\circ$

and $\angle C=4x=4\times 20^\circ=80^\circ$

Question: 9

A triangle ABC is right angled at A. L is a point on BC such that $AL\perp BC$. Prove that $\angle BAL=\angle ACB$.

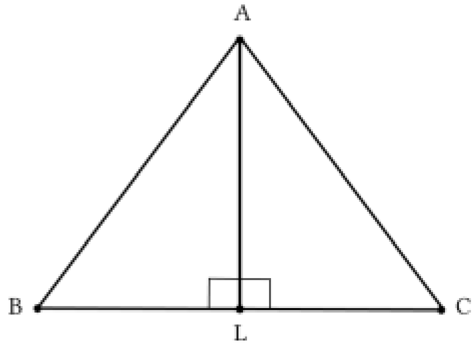
Solution:

Given:

In $\Delta ABC,\ \angle A=90^\circ$ and $AL\perp BC$

To prove:

$\angle BAL=\angle ACB$



Proof:

In ΔABC and ΔLAC,

$\angle BAC = \angle ALC$ ··· (i)

and $\angle ABC = \angle ABL$ ..... (ii) [Common angle]

Adding equations (i) and (ii), we get

$\angle BAC + \angle ABC = \angle ALC + \angle ABL$ ..... (iii)

In ΔABC, $\angle BAC + \angle ACB = \angle ABC = 180^{\circ}$

[The sum of the angles of a triangle is 180°.]

$\Rightarrow \angle BAC + \angle ABC = 180^{\circ} - \angle ACB$ ..... (iv)

In ΔABL, $\angle ABL + \angle ALB + \angle BAL = 180^{\circ}$

[The sum of the angles of a triangle is 180°]

$\Rightarrow \angle ABL + \angle ALC = 180^{\circ} - \angle BAL$ ..... (v)

$[\angle ALC = \angle ALB = 90^\circ]$

Substituting the value from equation (iv) and (v) in equation (iii), we get

$180^\circ - \angle ACB = 180^\circ - \angle BAL$

$\Rightarrow \angle ACB = \angle BAL$

Hence, it is proved.

Question: 10

Two lines are respectively perpendicular to two parallel lines. Show that they are parallel to each other.

Solution:

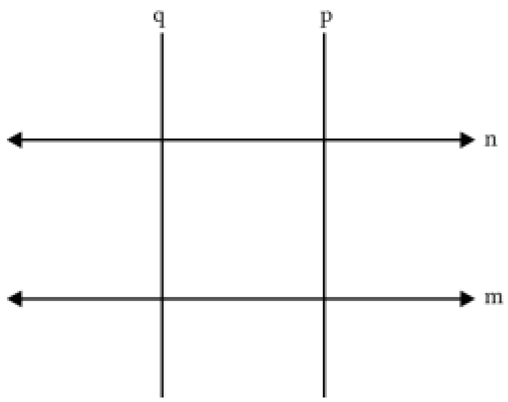
Given:

Two lines m and n are parallel and another two lines *p* and *q* are perpendicular to *m* and *n* respectively.

Or $p \perp m$ and $p \perp n$, $q \perp m$ and $q \perp n$

To prove:

$p \parallel q$



Proof:

Since, $m \parallel n$ and *p* is perpendicular to *m* and *n*.

So,

p is perpendicular to *m* ...(i)

p is perpendicular to *n* ...(ii)

Since, $m \mathbin{//} n$ and *q* is perpendicular to *m* and *n*.

So,

q is perpendicular to *m* ...(iii)

q is perpendicular to *n* ...(iv)

From the equations (i) and (iii) [Or from (ii) and (iv)], we get,

$p \parallel q$.

[If two lines are perpendicular to the same line, the lines are parallel to each other.]

Hence, it is $p \parallel q$.

Exercise: 6.4

Question: 1

If two lines intersect, prove that the vertically opposite angles are equal.

Solution:

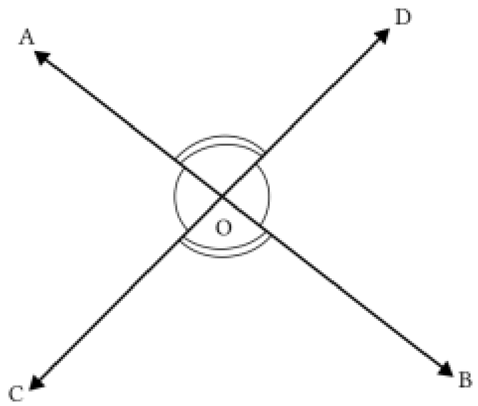
Given:

Two lines AB and CD intersect at point O.

To prove:

(i) $\angle AOC = \angle BOD$

(ii) $\angle AOD = \angle BOC$



Proof:

(i) Since, ray OA stands on line CD,

$\therefore \angle AOC + \angle AOD = 180^\circ$ ... (i)

[Linear pair axiom]

Similarly, ray OD stands on line AB.

$\therefore \angle AOD + \angle BOD = 180^\circ$ ... (ii)

From equations (i) and (ii), we get

$\angle AOC + \angle AOD = \angle AOD + \angle BOD$

$\Rightarrow \angle AOC = \angle BOD$

(ii) Since, ray OD stands on line AB,

$\therefore \angle AOD + \angle BOD = 180^\circ$ ... (iii)

[Linear pair axiom]

Similarly, ray OB stands on line CD.

$\therefore \angle DOB + \angle BOC = 180^\circ$ ... (iv)

From equations (iii) and (iv), we get

$\angle AOD + \angle BOD = \angle DOB + \angle BOC$

$\Rightarrow \angle AOD = \angle BOC$

Question: 2

Bisectors of interior $\angle B$ and exterior $\angle ACD$ of ΔABC intersect at the point T.

Prove that:

$$\angle BTC = \frac{1}{2}\angle BAC.$$

Solution:

Given:

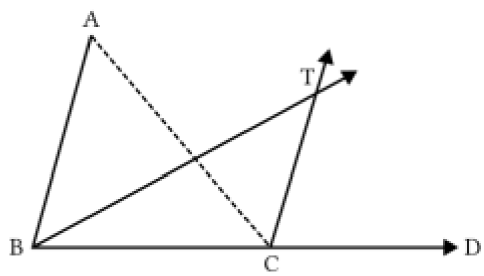
In ΔABC, the bisectors of $\angle ABC$ and $\angle ACD$ meet at point T.

Construction:

Produce BC to D.

To Prove:

$$\angle BTC = \frac{1}{2}\angle BAC$$



Proof:

In ΔABC, $\angle ACD$ is an exterior angle.

$\therefore \angle ACD = \angle ABC + \angle CAB$

[The exterior angle of a triangle is equal to the sum of two opposite interior angles.]

$\Rightarrow \frac{1}{2}\angle ACD = \frac{1}{2}\angle CAB + \frac{1}{2}\angle ABC$

[Dividing both sides by 2]

$\Rightarrow \angle TCD = \frac{1}{2}\angle CAB + \frac{1}{2}\angle ABC$ ... (i)

[$\because$ CT is a bisector of $\angle ACD \Rightarrow \frac{1}{2}\angle ACD = \angle TCD$]

In ΔBTC,

$\therefore \angle TCD = \angle BTC + \angle CBT$

[The exterior angle of a triangle is equal to the sum of two interior opposite angles.]

$\Rightarrow \angle TCD = \angle BTC + \frac{1}{2}\angle ABC$ ... (ii)

[$\because$ BT bisector of $\angle ABC \Rightarrow \angle CBT = \frac{1}{2}\angle ABC$]

From equations (i) and (ii), we get

$$\frac{1}{2}\angle CAB + \frac{1}{2}\angle ABC = \angle BTC + \frac{1}{2}\angle ABC$$

$$\Rightarrow \frac{1}{2}\angle CAB = \angle BTC$$

or $\frac{1}{2}\angle BAC = \angle BTC$

Hence, it is proved.

Question: 3

A transversal intersects two parallel lines. Prove that the bisectors of any pair of corresponding angles so formed are parallel.

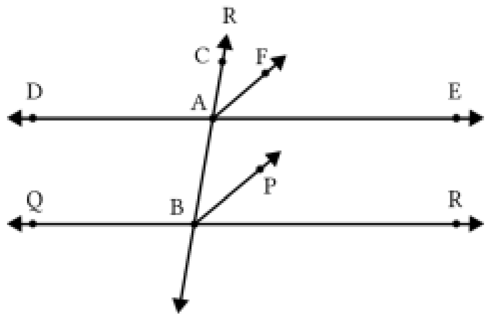
Solution:

Given:

Two lines DE and QR are parallel. A transversal intersects the lines DE and QR at A and B respectively. Also, BP and AF are the bisectors of angles $\angle ABR$ and $\angle CAE$ respectively.

To prove:

$BP \parallel AF$



Proof:

Given, $DE \parallel QR$

$\Rightarrow \angle CAE = \angle ABR$

[Corresponding angles]

$\Rightarrow \frac{1}{2}\angle CAE = \frac{1}{2}\angle ABR$

[Dividing both sides by 2]

$\Rightarrow \angle CAF = \angle ABP$

[$\because$ BP and AF are the bisectors of angles $\angle ABR$ and $\angle CAE$ respectively]

This implies that AF is parallel to BP as the corresponding angles $\angle CAF$ and $\angle ABP$ are equal.

Hence, $AF \parallel BP$.

Question: 4

Prove that through a given point, we can draw only one perpendicular on a given line.

[Hint: Use proof by contradiction].

Solution:

Given:

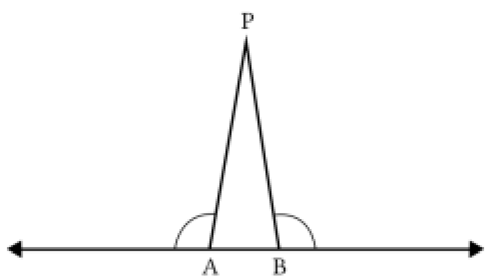
Consider a line *l* and a point P.

To prove:

Only one perpendicular can be drawn from P to *l*.

Construction:

Suppose that two lines PA and PB are passing through the point P and they are perpendicular to *l*.



Proof:

In ΔAPB,

$\angle PAB + \angle P + \angle PBA = 180^\circ$

[The angle sum property of a triangle]

$\Rightarrow 90^\circ + \angle P + 90^\circ = 180^\circ$ (since *PA* is perpendicular to

l and PB is perpendicular to *l*)

$\Rightarrow \angle P = 180^\circ - 180^\circ$

$\therefore \angle P = 0^\circ$

(Which is possible only when the lines PA and PB

coincide)

Hence, only one perpendicular line can be drawn through

a given point.

Question: 5

Prove that two lines that are respectively perpendicular to two intersecting lines intersect each other.

[Hint: Use proof by contradiction].

Solution:

Given:

Let lines *l* and m be two intersecting lines. Again, let *n* and *p* be another two lines which are perpendicular to the intersecting lines.

To prove:

Two lines *n* and *p* intersect at a point.

Proof:

Let us consider lines *n* and *p* are not intersecting, and then it means they are parallel to each other

i.e., $n \parallel p$ ... (i)

Since, lines *n* and *p* are perpendicular to *l* and *m* respectively, but from equation (i) $n \parallel p$, it is implied that $l \parallel m$, it is a contradiction.

Thus, our assumption is wrong.

Hence, lines *n* and *p* intersect at a point.

Question: 6

Prove that a triangle must have at least two acute angles.

Solution:

Given:

$\angle ABC$ is a triangle

To prove:

$\angle ABC$ must have two acute angles.

Proof:

Let us consider the following cases:

Case I:

When two angles are 90°:

Suppose two angles are $\angle B = 90^\circ$ and $\angle C = 90^\circ$.

The sum of all three angles of a triangle is 180°.

$\therefore \angle A + \angle B + \angle C = 180^\circ$

$\therefore \angle A + 90^\circ + 90^\circ = 180^\circ$

$\Rightarrow \angle A = 180^\circ - 180^\circ = 0$, which is not possible.

Hence, this case is rejected.

Case II:

When two angle are obtuse:

Suppose two angles $\angle B$ and $\angle C$ are more than 90°,

the sum of all three angles of a triangle is 180°.

$\therefore \angle A + \angle B + \angle C = 180^\circ$

$\angle A = 180^\circ - (\angle B + \angle C) = 180^\circ$ – (an angle greater than 180°)

$\angle A$ = negative angle, which is not possible.

Hence, this case is also rejected.

Case III:

When one angle is 90° and the other is obtuse:

Suppose $\angle B = 90^\circ$ and $\angle C$ is obtuse.

The sum of all three angles of a triangle is 180°.

$\therefore \angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow \angle A = 180^\circ - (90^\circ + \angle C)$

$= 90^\circ - \angle C$

= negative angle, which is not possible.

Hence, this case is also rejected.

Case IV:

When two angles are acute, then the sum of two angles is less than 180° so that the third angle is also acute.

Hence, a triangle must have at least two acute angles.

Question: 7

In Fig. 6.17, $\angle Q > \angle R$, PA is the bisector of $\angle QPR$ and $PM \perp QR$. Prove that

$$\angle APM = \frac{1}{2}(\angle Q - \angle R).$$

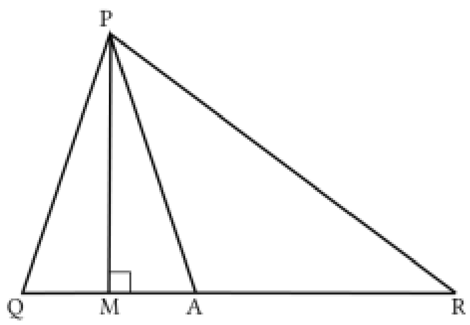


Fig. 6.17

Solution:

Given:

$\angle Q > \angle R$, PA is the bisector of $\angle QPR$ and $PM \perp QR$.

To prove:

$$\angle APM = \frac{1}{2}(\angle Q - \angle R).$$

Proof:

Since, PA is the bisector of $\angle QPR$

$\therefore \angle QPA = \angle APR$ ... (i)

In ΔPQM, $\angle Q + \angle PMQ + \angle QPM = 180^\circ$

[Angle sum property of triangles]

$\Rightarrow \angle Q + 90^\circ + \angle QPM = 180^\circ$

$[\angle PMQ = 90^\circ]$

$\Rightarrow \angle Q = 90^\circ - \angle QPM$ ... (ii)

In ΔPMR, $\angle PMR + \angle R + \angle RPM = 180^\circ$

[Angle sum property of triangles]

$\Rightarrow 90^\circ + \angle R + \angle RPM = 180^\circ$

$[\angle PMQ = 90^\circ]$

$\Rightarrow \angle R = 180^\circ - 90^\circ - \angle RPM$

$\Rightarrow \angle R = 90^\circ - \angle RPM$ ... (iii)

Subtracting equations (iii) from (ii), we get

$\angle Q - \angle R = (90^\circ - \angle QPM) - (90^\circ - \angle RPM)$

$\Rightarrow \angle Q - \angle R = \angle RPM - \angle QPM$

$\Rightarrow \angle Q - \angle R = (\angle RPA + \angle APM) - (\angle QPA + \angle APM) \ldots$ (iv)

$\Rightarrow \angle Q - \angle R = \angle QPA + \angle APM - \angle QPA + \angle APM$

[Using equation (i)]

$\Rightarrow \angle Q - \angle R = 2\angle APM$

$\therefore \angle APM = \frac{1}{2}(\angle Q - \angle R)$