NCERT Exemplar

Class 10 Maths

Chapter 3- Pair of Linear Equations in Two Variables

Exercise 3.1 (13)

Question 1

Graphically, the pair of equations

$6x-3y+10=0$

$2x-y+9=0$

Represents two lines which are

a. **intersecting at exactly one point**

b. **intersecting at exactly two points**

c. **coincident**

d. **parallel**

Solution:

(d)

Given equations are: $6x-3y+10=0$ and $2x-y+9=0$

Compare with $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$,

$\frac{a_1}{a_2}=\frac{6}{2}=\frac{3}{1}, \frac{b_1}{b_2}=\frac{-3}{-1}=\frac{3}{1}$ and $\frac{c_1}{c_2}=\frac{10}{9}$.

Now, observe that

$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

If $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$ holds then these are parallel lines.

Thus, the lines are parallel.

Question 2

The pair of equations $x+2y+5=0$ **and** $-3x-6y+1=0$ **have**

a. a unique solution

b. exactly two solutions

c. infinitely many solutions

d. no solution

Solution:

(d)

Given equations are

$x+2y+5=0$ and $-3x-6y+1=0$

Compare with $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$,

$\frac{a_1}{a_2}=\frac{1}{-3}=-\frac{1}{3}$, $\frac{b_1}{b_2}=\frac{2}{-6}=-\frac{1}{3}$ and $\frac{c_1}{c_2}=\frac{5}{1}$.

Now, observe that

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

If $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$ holds then the lines have no solution.

Thus, the lines have no solution.

Question 3

If a pair of linear equations is consistent, then the lines will be

a. parallel

b. always coincident

c. intersecting or coincident

d. always intersecting

Solution:

(c)

We have a given condition that when a pair of linear equations is consistent, then the lines are intersecting or coincident.

Thus, the lines are intersecting or coincident.

Question 4

The pair of equations $y=0$ and $y=-7$ has

a. **one solution**

b. **two solutions**

c. **infinitely many solutions**

d. **no solution**

Solution:

(d)

Given: $y=0$ and $y=-7$

The pair of linear equations $y=0$ and $y=-7$ both lines are parallel to x-axis.

Also, parallel lines never intersect.

Thus, there is no solution of the linear equations.

Question 5

The pair of equations $x=a$ and $y=b$ graphically represent lines which are

a. **parallel**

b. **intersecting at (b,a)**

c. **coincident**

d. **intersecting at (a,b)**

Solution:

(d)

Given: $x=a$ and $y=b$

The line $x=a$ is parallel to y–axis and the line $y=b$ is parallel to x–axis.

Thus, the lines $x=a$ and $y=b$ are perpendicular to each other and intersect each other at (a,b). Fgb

Question 6

For what value of k, do the equations $3x-y+8=0$ and $6x-ky=-16$ represent coincident lines?

a. $\frac{1}{2}$

b. $-\frac{1}{2}$

c. 2

d. -2

Solution:

(c)

Given equations are: $3x-y+8=0$ and $6x-ky=-16$

We can rewrite them as,

$3x-y+8=0$ and $6x-ky+16=0$

Compare with $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$,

$\frac{a_1}{a_2}=\frac{3}{6}=\frac{1}{2}, \frac{b_1}{b_2}=\frac{-1}{-k}=\frac{1}{k}$ and $\frac{c_1}{c_2}=\frac{8}{16}=\frac{1}{2}$.

For coincident lines,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{1}{2} = \frac{1}{k} = \frac{1}{2}$$

$$k = 2$$

Thus, the value of k is 2.

Question 7

If the lines given by $3x + 2ky = 2$ and $2x + 5y + 1 = 0$ are parallel, then the value of k is

a. $\frac{-5}{4}$

b. $\frac{2}{5}$

c. $\frac{15}{4}$

d. $\frac{3}{2}$

Solution:

(c)

Given equations are: $3x + 2ky = 2$ and $2x + 5y + 1 = 0$

We can rewrite them as,

$3x + 2ky - 2 = 0$ and $2x + 5y + 1 = 0$

Compare with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$,

$\frac{a_1}{a_2}=\frac{3}{2}$, $\frac{b_1}{b_2}=\frac{2k}{5}$ and $\frac{c_1}{c_2}=\frac{-2}{1}$.

For parallel lines,

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

$$\frac{3}{2}=\frac{2k}{5}\neq\frac{-2}{1}$$

Solve the first two ratios,

$$\frac{3}{2}=\frac{2k}{5}$$

$$15=4k$$

$$k=\frac{15}{4}$$

Thus, the value of k is $\frac{15}{4}$.

Question 8

The value of c for which the pair of equations $cx-y=2$ and $6x-2y=3$ will have infinitely many solutions is

a. 3

b. -3

c. -12

d. no value

Solution:

(d)

Given equations are: $cx-y=2$ and $6x-2y=3$

We can rewrite them as,

$cx - y - 2 = 0$ and $6x - 2y - 3 = 0$

Compare with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$,

$$\frac{a_1}{a_2} = \frac{c}{6}, \frac{b_1}{b_2} = \frac{-1}{-2} = \frac{1}{2} \text{ and } \frac{c_1}{c_2} = \frac{-2}{-3} = \frac{2}{3}$$

For infinite many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

By the above values,

$$\frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Thus, no value of c exists for which the equations have infinite solutions.

Question 9

One equation of a pair of dependent linear equations is $-5x + 7y = 2$. The second equation can be

a. $10x + 14y + 4 = 0$

b. $-10x - 14y + 4 = 0$

c. $-10x + 14y + 4 = 0$

d. $10x - 14y = -4$

Solution:

(d)

For dependent linear equations,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Given linear equation is $-5x+7y-2=0$.

For case (A), $10x+14y+4=0$

$$\frac{a_1}{a_2}=\frac{-5}{10}=-\frac{1}{2}, \frac{b_1}{b_2}=\frac{7}{14}=\frac{1}{2}$$

$$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$$

Thus, $10x+14y+4=0$ and $-5x+7y-2=0$ are not dependent.

For case (B), $-10x-14y+4=0$

$$\frac{a_1}{a_2}=\frac{-5}{-10}=\frac{1}{2}, \frac{b_1}{b_2}=\frac{7}{-14}=-\frac{1}{2}$$

$$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$$

Thus, $-10x-14y+4=0$ and $-5x+7y-2=0$ are not dependent.

For case (C), $-10x+14y+4=0$

$$\frac{a_1}{a_2}=\frac{-5}{-10}=\frac{1}{2}, \frac{b_1}{b_2}=\frac{7}{14}=\frac{1}{2} \text{ and } \frac{c_1}{c_2}=\frac{-2}{4}=-\frac{1}{2}$$

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

Thus, $-10x+14y+4=0$ and $-5x+7y-2=0$ are not dependent.

For case (D), $10x-14y+4=0$

$$\frac{a_1}{a_2}=\frac{-5}{10}=-\frac{1}{2}, \frac{b_1}{b_2}=\frac{7}{-14}=-\frac{1}{2} \text{ and } \frac{c_1}{c_2}=\frac{-2}{4}=-\frac{1}{2}$$

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

Thus, $10x - 14y + 4 = 0$ and $-5x + 7y - 2 = 0$ are dependent.

Question 10

A pair of linear equations which has a unique solution $x = 2, y = -3$ is

a. $x + y = -1$
$2x - 3y = -5$

b. $2x + 5y = -11$
$4x + 10y = -22$

c. $2x - y = 1$
$3x + 2y = 0$

d. $x - 4y - 14 = 0$
$5x - y - 13 = 0$

Solution:

(d)

For case (A),

Given equations are:

$x + y = -1$ ... (1)

$2x - 3y = -5$ ... (2)

Put $x = 2, y = -3$ in LHS of equation (1),

$x + y = 2 + (-3)$

$= 2 - 3$

$= -1$

$= \text{RHS}$

Put $x = 2, y = -3$ in LHS of equation (2),

$2x - 3y = 2(2) - 3(-3)$

$= 4 + 9$

$= 13$

$13 \neq -5$

$\text{LHS} \neq \text{RHS}$

Thus, $x = 2,\ y = -3$ satisfies only one of the equation.

Hence, case (A) is false.

For case (B),

Given equations are:

$2x + 5y = -11$ ... (1)

$4x + 10y = -22$ ... (2)

Put $x = 2,\ y = -3$ in LHS of equation (1),

$2x + 5y = 2(2) + 5(-3)$

$= 4 - 15$

$= -11$

$= \text{RHS}$

Put $x = 2,\ y = -3$ in LHS of equation (2),

$4x + 10y = 4(2) + 10(-3)$

$= 8 - 30$

$= -22$

$= \text{RHS}$

Thus, $x = 2,\ y = -3$ satisfies both the equations.

But these pair of equations are coincident.

So, there are infinitely many solutions.

Hence, case (B) is false.

For case (C),

Given equations are:

$2x - y = 1$ ... (1)

$3x + 2y = 0$ ... (2)

Put $x = 2,\ y = -3$ in LHS of equation (1),

$2x - y = 2(2) - (-3)$

$= 4 + 3$

$= 7$

$7 \neq -11$

$\text{LHS} \neq \text{RHS}$

Put $x = 2,\ y = -3$ in LHS of equation (2),

$3x + 2y = 3(2) + 2(-3)$

$= 6 - 6$

$= 0$

$= \text{RHS}$

Thus, $x = 2,\ y = -3$ satisfies only one of the equation.

Hence, case (C) is false.

For case (D),

Given equations are:

$x - 4y - 14 = 0$ ... (1)

$5x - y - 13 = 0$ ... (2)

Put $x = 2,\ y = -3$ in LHS of equation (1),

$x - 4y - 14 = (2) - 4(-3) - 14$

$= 2 + 12 - 14$

$= 0$

$= \text{RHS}$

Put $x = 2,\ y = -3$ in LHS of equation (2),

$5x-y-13=5(2)-(-3)-13$

$=10+3-13$

$=0$

$=\text{RHS}$

Thus, $x=2,\ y=-3$ satisfies both the equation.

Hence, case (D) is true.

Question 11

If $x=a,\ y=b$ is the solution for the equations, $x-y=2$ and $x+y=4$, then the values of a and b respectively, are

a. 3 and 5

b. 5 and 3

c. 3 and 1

d. -1 and -3

Solution:

(c)

If $x=a,\ y=b$ is the solution of the equations $x-y=2$ and $x+y=4$.

$a-b=2$ ... (1)

$a+b=4$ ... (2)

Add equations (1) and (2),

$a-b+a+b=2+4$

$2a=6$

$a=3$

Put the value of a in equation (2),

$a+b=4$

$3+b=4$

$b = 4 - 3$

$b = 1$

Thus, $a = 3$ and $b = 1$.

Question 12

Aruna has only Re 1 and Rs 2 coins with her. If the total number of coins she has is 50, and the amount of money with her is Rs 75, then the number of Re 1 and Rs 2 coins, respectively, are

a. 35 and 15

b. 35 and 20

c. 15 and 35

d. 25 and 25

Solution:

(d)

Let a be the number of coins of Re 1 and b be the number of coins Rs. 2.

Total number of coins $= 50$

$a + b = 50$ ... (1)

Total amount of money with her $= \text{Rs } 75$

$1 \times a + 2 \times b = 75$

$a + 2b = 75$ ... (2)

Subtract equation (1) from equation (2),

$a + 2b - (a + b) = 75 - 50$

$a + 2b - a - b = 25$

$b = 25$

Put the value of b in equation (1),

$a + b = 50$

$a+25=50$

$a=50-25$

Thus, $a=25$ and $b=25$.

Question 13

The father's age is six times his son's age. Four years hence, the age of the father will be four times his son's age. The present ages, in years, of the son and the father, respectively, are

a. 4 and 24

b. 5 and 30

c. 6 and 36

d. 3 and 24

Solution:

(d)

Let the father's age $=a$ years

Let the son's age $=b$ years

As the father's age is six times his son's age,

$a=6b$ ... (1)

After 4 years,

The father's age $=a+4$ years

The son's age $=b+4$ years

Four years hence, the age of the father will be four times his son's age,

$a+4=4(b+4)$

$a-4b=12$ ... (2)

From equation (1) and (2),

$6b-4b=12$

$2b = 12$

$b = 6$

Put the value of b in equation (1),

$a = 6 \times 6$

$= 36$

Thus, the father's age is 36 years and the son's age is 6 years.

Exercise 3.2 (6)

Question 1

Do the following pair of linear equations have no solution? Justify your answer.

(i) $2x + 4y = 3$

$12y + 6x = 6$

(ii) $x = 2y$

$y = 2x$

(iii) $3x + y - 3 = 0$

$2x + \frac{2}{3}y = 2$

Solution:

(i) The given equations can be written as

$2x + 4y - 3 = 0$

$12y + 6x - 6 = 0$

Compare with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$,

$a_1 = 2, b_1 = 4\ c_1 = -3$ and $a_2 = 6, b_2 = 12, c_2 = -6$

$$\frac{a_1}{a_2}=\frac{2}{6}=\frac{1}{3},\frac{b_1}{b_2}=\frac{4}{12}=\frac{1}{3},\frac{c_1}{c_2}=\frac{-3}{-6}=\frac{1}{2}$$

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

Thus, the pair of linear equations has no solution.

(ii) The given equations can be written as

$$x-2y=0$$

$$-2x+y=0$$

Compare with $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$,

$a_1=1, b_1=-2, c_1=0$ and $a_2=-2, b_2=1, c_2=0$

$$\frac{a_1}{a_2}=\frac{1}{-2},\frac{b_1}{b_2}=\frac{-2}{1}$$

$$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$$

Thus, the pair of linear equations has a unique solution.

(iii) The given equations can be written as

$$3x+y-3=0$$

$$2x+\frac{2}{3}y-2=0$$

Compare with $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$,

$a_1=3, b_1=1, c_1=-3$ and $a_2=2, b_2=\frac{2}{3}, c_2=-2$

$$\frac{a_1}{a_2}=\frac{3}{2},\frac{a_1}{a_2}=\frac{3}{2},\frac{c_1}{c_2}=\frac{-3}{-2}=\frac{3}{2}$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Thus, the pair of linear equations has infinitely many solutions.

Question 2

Do the following equations represent a pair of coincident lines? Justify your answer.

(i) $3x + \frac{1}{7}y = 3$

$7x + 3y = 7$

(ii) $-2x - 3y = 1$

$6y + 4x = -2$

(iii) $\frac{x}{2} + y + \frac{2}{5} = 0$

$4x + 8y + \frac{5}{16} = 0$

Solution:

(i) The given equations can be written as

$$3x + \frac{1}{7}y - 3 = 0$$

$$7x + 3y - 7 = 0$$

Compare with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$,

$$a_1 = 3, b_1 = \frac{1}{7}, c_1 = -3 \text{ and } a_2 = 7, b_2 = 3, \ c_2 = -7$$

$$\frac{a_1}{a_2}=\frac{3}{7},\ \frac{b_1}{b_2}=\frac{\frac{1}{7}}{3}=\frac{1}{7}\times\frac{1}{3}=\frac{1}{21}$$

$$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$$

Thus, the equations do not represent a pair of coincident lines.

(ii) The given equations can be written as

$$-2x-3y-1=0$$

$$4x+6y+2=0$$

Compare with $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$,

$a_1=-2, b_1=-3, c_1=-1$ and $a_2=4, b_2=6, c_2=2$

$$\frac{a_1}{a_2}=\frac{-2}{4}=\frac{-1}{2},\ \frac{b_1}{b_2}=\frac{-3}{6}=\frac{-1}{2},\ \frac{c_1}{c_2}=\frac{-1}{2}$$

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

Thus, the equations represent a pair of coincident lines.

(iii) The given equations can be written as

$$\frac{x}{2}+y+\frac{2}{5}=0$$

$$4x+8y+\frac{5}{16}=0$$

Compare with $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$.

$$a_1 = \frac{1}{2}, b_1 = 1, c_1 = \frac{2}{5} \text{ and } a_2 = 4, b_2 = 8, c_2 = \frac{5}{16}$$

$$\frac{a_1}{a_2} = \frac{\frac{1}{2}}{4} = \frac{1 \times 1}{2 \times 4} = \frac{1}{8}, \frac{b_1}{b_2} = \frac{1}{8}, \frac{c_1}{c_2} = \frac{\frac{2}{5}}{\frac{5}{16}} = \frac{2}{5} \times \frac{16}{5} = \frac{32}{25}$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Thus, the equations do not represent a pair of coincident lines.

Question 3

Are the following pair of linear equations consistent? Justify your answer.

(i) $-3x - 4y = 12$

$4y + 3x = 12$

(ii) $\frac{3}{5}x - y = \frac{1}{2}$

$\frac{1}{5}x - 3y = \frac{1}{6}$

(iii) $2ax + by = a;\ 4ax + 2by - 2a = 0;\ a, b \neq 0$

(iv) $x + 3y = 11$

$2(2x + 6y) = 22$

Solution:

(i) The given equations can be written as

$-3x - 4y - 12 = 0$

$3x + 4y - 12 = 0$

Compare with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$.

$a_1 = -3, b_1 = -4, c_1 = -12$ and $a_2 = 3, b_2 = 4, c_2 = -12$

$$\frac{a_1}{a_2} = \frac{-3}{3} = -1, \frac{b_1}{b_2} = \frac{-4}{4} = -1, \frac{c_1}{c_2} = \frac{-12}{-12} = 1$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Thus, the pair of linear equations is inconsistent.

(ii) The given equations can be written as

$$\frac{3}{5}x - y - \frac{1}{2} = 0$$

$$\frac{1}{5}x - 3y - \frac{1}{6} = 0$$

Compare with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$.

$a_1 = \frac{3}{5}, b_1 = -1, c_1 = -\frac{1}{2}$ and $a_2 = \frac{1}{5}, b_2 = -3, c_2 = -\frac{1}{6}$

$$\frac{a_1}{a_2} = \frac{\frac{3}{5}}{\frac{1}{5}} = \frac{3}{5} \times \frac{5}{1} = \frac{3}{1}, \frac{b_1}{b_2} = \frac{-1}{-3} = \frac{1}{3}$$

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Thus, the pair of linear equations is consistent.

(iii) The given equations can be written as

$$2ax + by - a = 0$$

$4ax + 2by - 2a = 0$

Compare with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$.

$a_1 = 2a, b_1 = b, c_1 = -a$ and $a_2 = 4a, b_2 = 2b, c_2 = -2a$

$$\frac{a_1}{a_2} = \frac{2a}{4a} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{b}{2b} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-a}{-2a} = \frac{1}{2}$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Thus, the pair of linear equations is consistent.

(iv) The given equations can be written as

$x + 3y - 11 = 0$

$4x + 12y - 22 = 0$

Compare with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$.

$a_1 = 1, b_1 = 3, c_1 = -11$ and $a_2 = 4, b_2 = 12, c_2 = -22$

$$\frac{a_1}{a_2} = \frac{1}{4}, \frac{b_1}{b_2} = \frac{3}{12} = \frac{1}{4}, \frac{c_1}{c_2} = \frac{-11}{-22} = \frac{1}{2}$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Thus, the pair of linear equations is inconsistent.

Question 4

For the pair of equations

$\lambda x + 3y = -7$

$2x + 6y = 14$

to have infinitely many solutions, the value of λ should be 1. Is the statement true? Give reasons.

Solution:

The given equations can be written as

$$\lambda x+3y+7=0$$

$$2x+6y-14=0$$

Compare with $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$.

$a_1=\lambda, b_1=3, c_1=7$ and $a_2=2, b_2=6, c_2=-14$

$$\frac{a_1}{a_2}=\frac{\lambda}{2}, \frac{b_1}{b_2}=\frac{3}{6}=\frac{1}{2}, \frac{c_1}{c_2}=\frac{7}{-14}=\frac{1}{-2}$$

Now, the given pair of linear equations will have infinitely many solutions if

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

So, $\frac{\lambda}{2}=\frac{1}{2}=\frac{-1}{2}$

$\lambda=1$ and $\lambda=-1$

But $1\neq-1$

Thus, the given statement is not true.

Question 5

For all real values of c, the pair of equations

$$x-2y=8$$

$$5x-10y=c$$

has a unique solution. Justify whether it is true or false.

Solution:

The given equations can be written as

$x-2y-8=0$

$5x-10y-c=0$

Compare with $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$.

$a_1=1, b_1=-2, c_1=-8$ and $a_2=5, b_2=-10, c_2=-c$

$\frac{a_1}{a_2}=\frac{1}{5}, \frac{b_1}{b_2}=\frac{-2}{-10}=\frac{1}{5}$

$\frac{a_1}{a_2}=\frac{b_1}{b_2}$

The equations do not have a unique solution.

Thus, the given statement is false.

Question 6

The line represented by $x=7$ is parallel to the x–axis. Justify whether the statement is true or not.

Solution:

Given: $x=7$

Any line which is parallel to x-axis is of the form $y=a$, where a is a real number.

So, $x=7$ is a straight line that is parallel to y-axis.

Thus, the statement is false.

Exercise 3.3 (22)

Question 1

For which value(s) of λ, do the pair of linear equations

$\lambda x + y = \lambda^2$ and $x + \lambda y = 1$ have

(i) no solution?

(ii) infinitely many solutions?

(iii) a unique solution?

Solution:

Given pair of linear equations:

$\lambda x + y = \lambda^2$ and $x + \lambda y = 1$

We can rewrite them as,

$\lambda x + y - \lambda^2 = 0$ and $x + \lambda y - 1 = 0$

Compare with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$.

Here, $a_1 = \lambda, b_1 = 1, c_1 = -\lambda^2$ and $a_2 = 1, b_2 = \lambda, c_2 = -1$.

$$\frac{a_1}{a_2} = \frac{\lambda}{1}, \frac{b_1}{b_2} = \frac{1}{\lambda}, \frac{c_1}{c_2} = \frac{-\lambda^2}{-1} = \frac{\lambda^2}{1}$$

(i) The given pair of linear equations has no solution if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}\ \frac{a_1}{a_2} = \frac{b_1}{b_2} \text{ and } \frac{a_1}{a_2} \neq \frac{c_1}{c_2}$$

Substitute the value of a_1, a_2, b_1, b_2, c_1 and c_2,

$$\frac{\lambda}{1} = \frac{1}{\lambda} \text{ and } \frac{\lambda}{1} \neq \frac{\lambda^2}{1}$$

$\lambda^2 = 1$ and $\lambda \neq \lambda^2$

$\lambda^2 = 1$ and $\lambda^2 - \lambda \neq 0$

$\lambda^2 = 1$ and $\lambda(\lambda - 1) \neq 0$

$\lambda = 1, -1$ and $\lambda \neq 0$

$\lambda = -1$

Hence, the given pair of linear equations will have no solution when $\lambda = -1$.

(ii) The given pair of linear equations will have infinitely many solutions if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{\lambda}{1} = \frac{1}{\lambda} = \frac{\lambda^2}{1}$$

$$\frac{\lambda}{1} = \frac{1}{\lambda} \text{ and } \frac{\lambda}{1} = \frac{\lambda^2}{1}$$

$\lambda^2 = 1$ and $\lambda^2 - \lambda = 0$

$\lambda = 1, -1$ and $\lambda(\lambda - 1) = 0$

$\lambda = 1, -1$ and $\lambda = 0, 1$

$\lambda = 1$

Hence, the given pair of linear equations will have infinitely many solutions when $\lambda = 1$.

(iii) The given pair of linear equations will have a unique solution if

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\frac{\lambda}{1} \neq \frac{1}{\lambda}$$

$\lambda^2 \neq 1$

$\lambda \neq 1, -1$

Hence, the given pair of linear equations will have a unique solution for all real values of λ except $1, -1$.

Question 2

For which value(s) of k will the pair of equations

$kx + 3y = k - 3$

$12x + ky = k$

have no solution?

Solution:

Given:

$kx + 3y = k - 3$

$12x + ky = k$

We can rewrite as,

$kx + 3y - (k - 3) = 0$

$12x + ky - k = 0$

Here, $a_1 = k, b_1 = 3, c_1 = -(k - 3)$ and $a_2 = 12, b_2 = k, c_2 = -k$

Now, the given pair of linear equations has no solution if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\frac{k}{12} = \frac{3}{k} \neq \frac{-(k-3)}{-k}$$

$$\frac{k}{12} = \frac{3}{k} \neq \frac{(k-3)}{k}$$

$$\frac{k}{12} = \frac{3}{k} \text{ and } \frac{3}{k} \neq \frac{(k-3)}{k}$$

$k^2 = 36$ and $3 \neq k - 3$

$k = \pm 6$ and $k = \pm 6$

$k = -6$

Hence, the given pair of linear equations has no solution when $k = -6$.

Question 3

For which values of a and b, will the following pair of linear equations have infinitely many solutions?

$$x + 2y = 1$$

$$(a-b)x + (a+b)y = a+b-2$$

Solution:

The given pair of linear equations can be written as

$$x + 2y - 1 = 0$$

$$(a-b)x + (a+b)y - (a+b-2) = 0$$

The given pair of linear equations has infinitely many solutions if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{1}{a-b} = \frac{2}{a+b} = \frac{-1}{-(a+b-2)}$$

$$\frac{1}{a-b} = \frac{2}{a+b} = \frac{1}{(a+b-2)}$$

So,

$$\frac{1}{a-b} = \frac{2}{a+b}$$

$$a + b = 2(a-b)$$

$$a + b = 2a - 2b$$

$a = 3b$ ......(1)

$\frac{2}{a+b} = \frac{1}{a+b-2}$

$2a + 2b - 4 = a + b$

$a + b = 4$ ......(2)

From (1) and (2),

$3b + b = 4$

$4b = 4$

$b = 1$

Substitute the value of b in (1),

$a = 3b$

$= 3 \times 1$

$= 3$

Thus, the pair of linear equations has infinitely many solutions if $a = 3, b = 1$.

Question 4

Find the value(s) of p in (i) to (iv) and p and q in (v) for the following pair of equations:

(i) $3x - y - 5 = 0$ **and** $6x - 2y - p = 0$,

if the lines represented by these equations are parallel.

(ii) $-x + py = 1$ **and** $px - y = 1$,

if the pair of equations has no solution.

(iii) $-3x + 5y = 7$ **and** $2px - 3y = 1$,

if the lines represented by these equations are intersecting at a unique point.

(iv) $2x + 3y - 5 = 0$ **and** $px - 6y - 8 = 0$,

if the pair of equations has a unique solution.

(v) $2x+3y=7$ and $2px+py=28-qy$,

if the pair of equations has infinitely many solutions.

Solution:

(i) The given pair of linear equations is

$3x-y-5=0$ and $6x-2y-p=0$

Here, $a_1=3, b_1=-1, c_1=-5$ and $a_2=6, b_2=-2, c_2=-p$

Now, the lines represented by the given pair of linear equations will be parallel if

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

$$\frac{3}{6}=\frac{-1}{-2}\neq\frac{-5}{-p}$$

$$\frac{1}{2}=\frac{1}{2}\neq\frac{5}{p}$$

$$\frac{1}{2}\neq\frac{5}{p}$$

$$p\neq 10$$

Hence, the lines represented by the given pair of linear equations will be parallel for all real values of p except 10.

(ii) The given pair of linear equations is $-x+py=1$ and $px-y=1$

We can rewrite them as,

$-x+py-1=0$ and $px-y-1=0$

Here, $a_1=-1, b_1=p, c_1=-1$ and $a_2=p, b_2=-1, c_2=-1$

Now, the lines represented by the given pair of linear equations will have no solution if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \frac{-1}{p} = \frac{p}{-1} \neq \frac{-1}{-1} \frac{-1}{p} = \frac{p}{-1} \neq \frac{1}{1}$$

$$\frac{-1}{p} = \frac{p}{-1}$$

$$p^2 = 1$$

$$p = \pm 1$$

If $p = 1$,

$$\frac{a_1}{a_2} = -1, \frac{b_1}{b_2} = -1, \frac{c_1}{c_2} = 1$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

If $p = -1$,

$$\frac{a_1}{a_2} = 1, \frac{b_1}{b_2} = 1, \frac{c_1}{c_2} = 1$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Hence, the lines represented by the given pair of linear equations will have no solution for all real values of p except 1.

(iii) The given pair of linear equations is $-3x + 5y = 7$ and $2px - 3y = 1$

We can rewrite them as,

$-3x + 5y - 7 = 0$ and $2px - 3y - 1 = 0$

Here, $a_1 = -3, b_1 = 5, c_1 = -7$ and $a_2 = 2p, b_2 = -3, c_2 = -1$

Now, the lines represented by the given pair of linear equations will intersect at a unique point if

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\frac{-3}{2p} \neq \frac{5}{-3}$$

$$10p \neq 9$$

$$p \neq \frac{9}{10}$$

Hence, the lines represented by the given pair of linear equations will intersect at a unique point for all real values of p except $\frac{9}{10}$.

(iv) The given pair of linear equations is
$2x+3y-5=0$ and $px-6y-8=0$

Here, $a_1 = 2, b_1 = 3, c_1 = -5$ and $a_2 = p, b_2 = -6, c_2 = -8$

Now, the lines represented by the given pair of linear equations will have a unique solution if

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\frac{2}{p} \neq \frac{3}{-6}$$

$$3p \neq -12$$

$$p \neq -4$$

Hence, the lines represented by the given pair of linear equations will have a unique solution for all real values of p except -4.

(v) The given pair of linear equations is $2x+3y=7$ and
$2px + py = 28 - qy$

We can rewrite them as,

$2x+3y-7=0$ and $2px+(p+q)y-28=0$

Here, $a_1=2, b_1=3, c_1=-7$ and $a_2=2p, b_2=p+q, c_2=-28$

Now, the lines represented by the given pair of linear equations will have infinitely many solutions if

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

$$\frac{2}{2p}=\frac{3}{p+q}=\frac{-7}{-28}$$

$$\frac{1}{p}=\frac{3}{p+q}=\frac{1}{4}$$

$$\frac{1}{p}=\frac{1}{4} \text{ and } \frac{3}{p+q}=\frac{1}{4}$$

$p=4$ and $p+q=12$

$p=4$ and $4+q=12$

$p=4$ and $p=4$

$q=12-4$ and $q=8$

Hence, the lines represented by the given pair of linear equations will have infinitely many solutions for all real values of p and q except 4 and 8.

Question 5

Two straight paths are represented by the equations $x-3y=2$ and $-2x+6y=5$. Check whether the paths cross each other or not.

Solution:

Given equations:

$x-3y=2$ and $-2x+6y=5$

We can rewrite them as,

$x-3y-2=0$ and $-2x+6y-5=0$

Here, $a_1=1, b_1=-3, c_1=-2$ and $a_2=-2, b_2=6, c_2=-5$

$$\frac{a_1}{a_2}=\frac{1}{-2}, \frac{b_1}{b_2}=\frac{-3}{6}, \frac{c_1}{c_2}=\frac{-2}{-5}$$

$$\frac{a_1}{a_2}=\frac{1}{2}, \frac{b_1}{b_2}=-\frac{1}{2}, \frac{c_1}{c_2}=\frac{2}{5}$$

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

Thus, the paths do not cross each other.

Question 6

Write a pair of linear equations which has the unique solution $x=-1, y=3$. How many such pairs can you write?

Solution:

Given: $x=-1, y=3$

Required pair of equations:

$x+y=2$

$2x+y=1$

There is infinite many such pair are possible.

Question 7

If $2x+y=23$ and $4x-y=19$, find the values of $5y-2x$ and $\frac{y}{x}-2$.

Solution:

Given equations:

$$2x + y = 23 \qquad ......(1)$$

$$4x - y = 19 \qquad ......(2)$$

Add equations (1) and (2),

$$2x + y + 4x - y = 23 + 19$$

$$6x = 42$$

$$x = 7$$

Put the value of x in equation (1),

$$2x + y = 23$$

$$2(7) + y = 23$$

$$14 + y = 23$$

$$y = 23 - 14$$

$$y = 9$$

Now, calculate the value of $5y - 2x$ and $\frac{y}{x} - 2$.

$$5y - 2x = 5(9) - 2(7)$$

$$= 45 - 14$$

$$= 31$$

$$\frac{y}{x} - 2 = \frac{9}{7} - 2$$

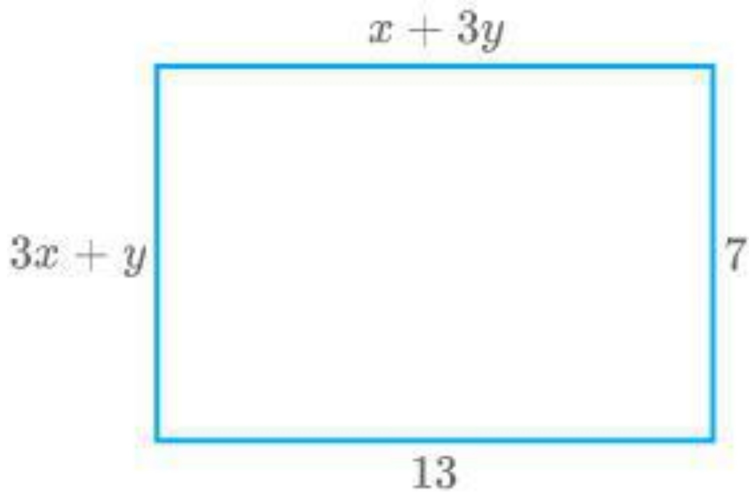
$$= \frac{9 - 14}{7}$$

$$= \frac{-5}{7}$$

Thus, $5y-2x=31$ and $\frac{y}{x}-2=\frac{-5}{7}$.

Question 8

Find the values of x and y in the following rectangle.



Solution:

The opposite sides of a rectangle are equal.

$3x+y=7$ ......(1)

$x+3y=13$ ......(2)

Multiply equation (2) by 3,

$3x+9y=39$ ......(3)

Subtract equation (1) from equation (3),

$3x+9y-(3x+y)=39-7$

$3x+9y-3x-y=39-7$

$8y=32$

$y=4$

Put the value of y in equation (1),

$3x + y = 7$

$3x + 4 = 7$

$3x = 7 - 4$

$3x = 3$

$x = 1$

Thus, $x = 1, y = 4$.

Question 9

Solve the following pairs of equations:

(i) $x + y = 3.3$

$\frac{0.6}{3x - 2y} = -1, 3x - 2y \neq 0$

(ii) $\frac{x}{3} + \frac{y}{4} = 4$

$\frac{5x}{6} - \frac{y}{8} = 4$

(iii) $4x + \frac{6}{y} = 15$

$6x - \frac{8}{y} = 14, y \neq 0$

(iv) $\frac{1}{2x} - \frac{1}{y} = -1$

$\frac{1}{x} + \frac{1}{2y} = 8, x, y \neq 0$

(v) $43x + 67y = -24$

$67x + 43y = 24$

(vi) $\frac{x}{a} + \frac{y}{b} = a + b$

$\frac{x}{a^2} + \frac{y}{b^2} = 2, a, b \neq 0$

(vii) $\frac{2xy}{x+y} = \frac{3}{2}\ \frac{xy}{2x-y} = \frac{-3}{10}, \quad x + y \neq 0, 2x - y \neq 0$

Solution:

(i) Given equations:

$x + y = 3.3 \quad \text{......(1)}$

$\frac{0.6}{3x - 2y} = -1$

$3x - 2y = -0.6 \quad \text{......(2)}$

Multiply equation (1) by 2,

$2x + 2y = 6.6 \quad \text{......(3)}$

Add equation (2) and (3),

$2x + 2y + 3x - 2y = 6.6 - 0.6$

$5x = 6$

$x = \frac{6}{5}$

$x = 1.2$

Put the value of x in equation (1),

$x + y = 3.3$

$1.2 + y = 3.3$

$y = 3.3 - 1.2$

$y = 2.1$

Thus, $x = 1.2, y = 2.1.$

(ii) Given equations:

$$\frac{x}{3} + \frac{y}{4} = 4 \qquad ......(1)$$

$$\frac{5x}{6} - \frac{y}{8} = 4 \qquad ......(2)$$

Multiply equation (1) by 12,

$$\frac{12x}{3} + \frac{12y}{4} = 48$$

$$4x + 3y = 48 \qquad ......(3)$$

Multiply equation (2) by 24,

$$\frac{(5 \times 24)x}{6} - \frac{24y}{8} = 96$$

$$20x - 3y = 96 \qquad ....(4)$$

Add equation (3) and (4),

$$4x + 3y + 20x - 3y = 48 + 96$$

$$24x = 144$$

$$x = 6$$

Put the value of x in equation (3),

$$4x + 3y = 48$$

$$4(6) + 3y = 48$$

$$24 + 3y = 48$$

$$3y = 48 - 24$$

$$3y = 24$$

$y = 8$

Thus, $x = 6, y = 8$.

(iii) Given equations:

$$4x + \frac{6}{y} = 15$$

$$6x - \frac{8}{y} = 14$$

Let $\frac{1}{y} = a$.

$4x + 6a = 15$ ......(1)

$6x - 8a = 14$ ......(2)

Multiply equation (1) by 4,

$16x + 24a = 60$ ......(3)

Multiply equation (2) by 3,

$18x - 24a = 42$ ......(4)

Add equation (3) and (4),

$16x + 24a + 18x - 24a = 60 + 42$

$34x = 102$

$x = 3$

Put the value of x in equation (1),

$4x + 6a = 15$

$4 \times 3 + 6a = 15$

$12 + 6a = 15$

$6a = 15 - 12$

$6a = 3$

$a = \frac{3}{6}$

$a = \frac{1}{2}$

Since $\frac{1}{y} = a$.

So, $\frac{1}{y} = \frac{1}{2}$

$y = 2$

Thus, $x = 3, y = 2$.

(iv) Given equations:

$\frac{1}{2x} - \frac{1}{y} = -1$

$\frac{1}{x} + \frac{1}{2y} = 8$

Put $\frac{1}{x} = a,\ \frac{1}{y} = b$.

$\frac{a}{2} - b = -1 \quad \text{......(1)}$

$a + \frac{b}{2} = 8 \quad \text{......(2)}$

Multiply the equation (1) by $\frac{1}{2}$,

$\frac{a}{4} - \frac{b}{2} = \frac{-1}{2} \quad \text{......(3)}$

Add equation (2) and (3),

$$a+\frac{b}{2}+\frac{a}{4}-\frac{b}{2}=8+\frac{-1}{2}$$

$$a+\frac{a}{4}=\frac{16-1}{2}$$

$$\frac{5}{4}a=\frac{15}{2}$$

$$a=\frac{15}{2}\times\frac{4}{5}$$

$$a=6$$

Put the value of a in equation (2),

$$a+\frac{b}{2}=8$$

$$6+\frac{b}{2}=8$$

$$\frac{b}{2}=8-6$$

$$\frac{b}{2}=2$$

$$b=4$$

So, $a=6, b=4$

$$\frac{1}{x}=6, \frac{1}{y}=4$$

$$x=\frac{1}{6}, y=\frac{1}{4}$$

Thus, $x=\frac{1}{6}, y=\frac{1}{4}$.

(v) Given equations:

$43x+67y=-24$ ......(1)

$67x+43y=24$ ......(2)

Add equation (1) and (2),

$110x+110y=0$

$x+y=0$ ......(3)

Subtract equation (1) and (2),

$-24x+24y=-48$

$-x+y=-2$ ......(4)

Add (3) and (4),

$x+y+(-x+y)=0+(-2)$

$x+y-x+y=-2$

$2y=-2$

$y=-1$

Put the value of y in equation (3),

$x+y=0$

$x-1=0$

$x=1$

Thus, $x=1, y=-1.$

(vi) Given equations:

$\frac{x}{a}+\frac{y}{b}=a+b$ ......(1)

$$\frac{x}{a^2}+\frac{y}{b^2}=2 \qquad ......(2)$$

Multiply the equation (1) by $\frac{1}{a}$,

$$\frac{x}{a^2}+\frac{y}{ab}=\frac{a+b}{a} \qquad ......(3)$$

Subtract equation (2) from (3),

$$\frac{x}{a^2}+\frac{y}{ab}-\left(\frac{x}{a^2}+\frac{y}{b^2}\right)=\frac{a+b}{a}-2\;\frac{x}{a^2}+\frac{y}{ab}-\frac{x}{a^2}-\frac{y}{b^2}=\frac{a+b}{a}-2$$

$$\frac{y}{ab}-\frac{y}{b^2}=\frac{a+b}{a}-2$$

$$y\left(\frac{1}{ab}-\frac{1}{b^2}\right)=\frac{a+b-2a}{a}$$

$$y\left(\frac{b-a}{ab^2}\right)=\frac{b-a}{a}$$

$$y=\frac{(b-a)}{a}\times\frac{ab^2}{(b-a)}$$

$$y=b^2$$

Put the value of y in equation (2),

$$\frac{x}{a^2}+\frac{y}{b^2}=2$$

$$\frac{x}{a^2}+\frac{b^2}{b^2}=2$$

$$\frac{x}{a^2}+1=2$$

$$\frac{x}{a^2} = 2 - 1$$

$$\frac{x}{a^2} = 1$$

$$x = a^2$$

Thus, $x = a^2, y = b^2$.

(vii) Given equations:

$$\frac{2xy}{x+y} = \frac{3}{2} \quad ......(1)$$

$$\frac{xy}{2x-y} = \frac{-3}{10} \quad ......(2)$$

$$3x + 3y = 4xy \quad ......(3)$$

$$10xy = -3(2x - y)$$

$$-6x + 3y = 10xy \quad ......(4)$$

Subtract equation (4) from (3),

$$3x + 3y - (-6x + 3y) = 4xy - 10xy$$

$$3x + 3y + 6x - 3y = 4xy - 10xy$$

$$9x = -6xy$$

$$-\frac{9}{6} = y$$

$$y = -\frac{3}{2}$$

Put the value of y in equation (3),

$$3x + 3y = 4xy$$

$$3x+3\left(-\frac{3}{2}\right)=4x\left(-\frac{3}{2}\right)$$

$$3x-\frac{9}{2}=-6x$$

$$3x+6x=\frac{9}{2}$$

$$9x=\frac{9}{2}$$

$$x=\frac{1}{2}$$

Thus, $x=\frac{1}{2}, y=-\frac{3}{2}$.

Put $x=0$ in both equations,

$y=0$.

So, $x=0, y=0$ is a solution if $x\neq 0, y\neq 0$.

Question 10

Find the solution of the pair of equations $\frac{x}{10}+\frac{y}{5}-1=0$ and $\frac{x}{8}+\frac{y}{6}=15$, hence, find λ, if $y=\lambda x+5$.

Solution:

Given equations:

$$\frac{x}{10}+\frac{y}{5}-1=0 \qquad \text{......(1)}$$

$$\frac{x}{8}+\frac{y}{6}=15 \qquad \text{......(2)}$$

Multiply equation (1) by 10,

$$\frac{10x}{10}+\frac{10y}{5}=10$$

$$x+2y=10 \qquad ......(3)$$

Multiply equation (2) by 24,

$$\frac{24x}{8}+\frac{24y}{6}=15\times 24$$

$$3x+4y=360 \qquad ......(4)$$

Multiply equation (3) by 3,

$$3x+6y=30 \qquad ......(5)$$

Subtract equation (4) from (5),

$$3x+6y-(3x+4y)=30-360$$

$$3x+6y-3x-4y=30-360$$

$$2y=-330$$

$$y=-165$$

Put the value of y in equation (5),

$$3x+6y=30$$

$$3x+6(-165)=30$$

$$3x-990=30$$

$$3x=30+990$$

$$3x=1020$$

$$x=340$$

Thus, $x=340,\ y=-165$.

Now, $y=\lambda x+5$

$$(-165)=\lambda(340)+5$$

$$-165-5=340\lambda$$

$$340\lambda=-170$$

$$\lambda=-\frac{170}{340}$$

$$\lambda=-\frac{1}{2}$$

Thus, $\lambda=-\frac{1}{2}$.

Question 11

By the graphical method, find whether the following pair of equations are consistent or not. If consistent, solve them.

(i) $3x+y+4=0$

$6x-2y+4=0$

(ii) $x-2y=6$

$3x-6y=0$

(iii) $x+y=3$

$3x+3y=9$

Solution:

(i) Given equations:

$3x+y+4=0$

$6x-2y+4=0$

Equation 1:

$3x+y+4=0$ ......(1)

Equation 2:

$6x - 2y + 4 = 0$ ......(2)

To represent these equations graphically, you must have at least two solutions for each equation.

For equation 1:

The points are $(0, -4)$ and $(-2, 2)$.

$y = -3x - 4$

x	0	-1	-2
y	-4	-1	2

For equation 2:

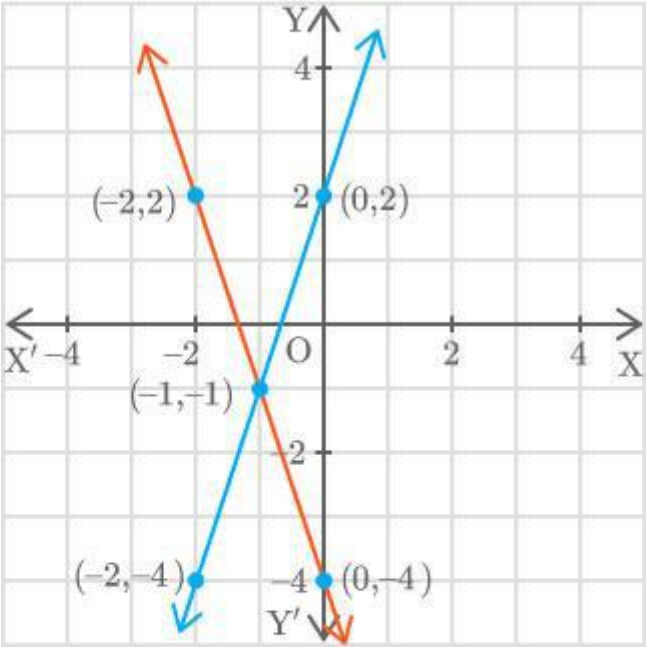
The points are $(0, 2)$ and $(-2, -4)$.

$6x - 2y + 4 = 0$

$-2y = -6x - 4$

$y = 3x + 2$

x	0	-1	-2
y	2	-1	-4



These lines intersect at point $(-1,-1)$. So, these lines are consistent.

(ii) Given equations:

$x-2y=6$

$3x-6y=0$

Equation 1:

$x-2y=6 \qquad ......(1)$

Equation 2:

$3x-6y=0 \qquad ......(2)$

To represent these equations graphically, you must have at least two solutions for each equation.

For equation 1:

The points are $(0,-3)$ and $(6,0)$.

$$y=\frac{x-6}{2}$$

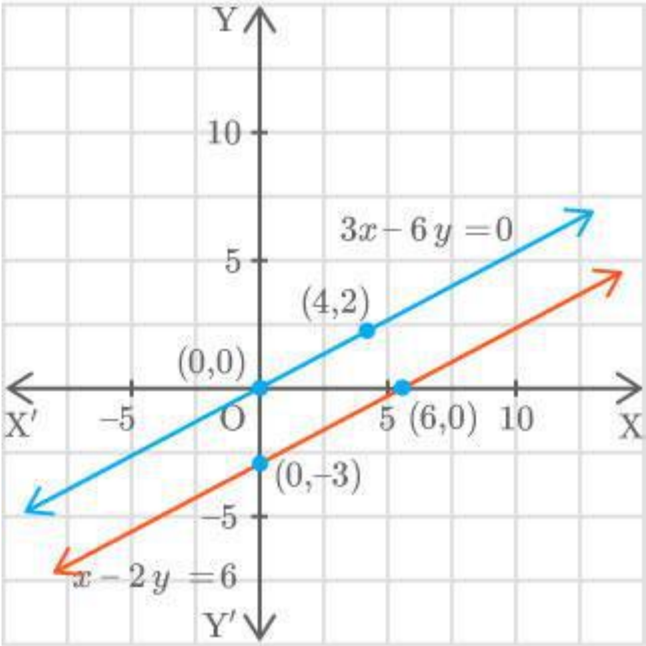
x	0	6
y	–3	0

For equation 2:

The points are $(0,0)$ and $(4,2)$.

$$y=\frac{x}{2}$$

x	0	4
y	0	2



These lines are parallel. So, these lines are inconsistent.

(iii) Given equations:

$x+y=3$

$3x+3y=9$

Equation 1:

$x+y=3$ ......(1)

Equation 2:

$3x+3y=9$ ......(2)

... (2)

To represent these equations graphically, you must have at least two solutions for each equation.

For equation 1:

The points are $(0,3)$ and $(3,0)$.

$y=3-x$

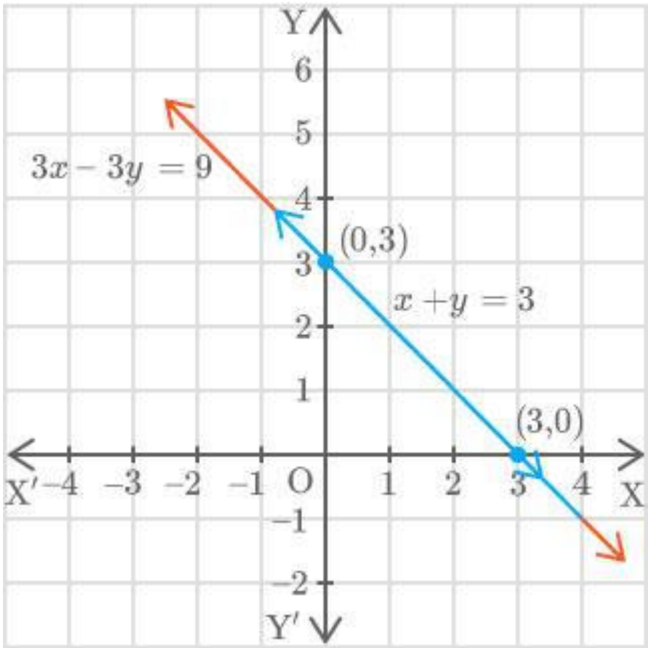
x	0	3
y	3	0

For equation 2:

The points are $(0,3)$ and $(3,0)$.

$y=3-x$

x	0	3
y	3	0



These lines are coinciding each other. So, these lines are consistent.

Question 12

Draw the graph for a pair of equations, $2x+y=4$ and $2x-y=4$. Write the vertices of the triangle formed by these lines and the y-axis. Also, find the area of this triangle.

Solution:

Equation 1:

$2x+y=4$ ......(1)

Equation 2:

$2x-y=4$ ......(2)

To represent these equations graphically, you must have at least two solutions for each equation.

For equation 1:

The points are $(0,4)$ and $(2,0)$.

$y=4-2x$

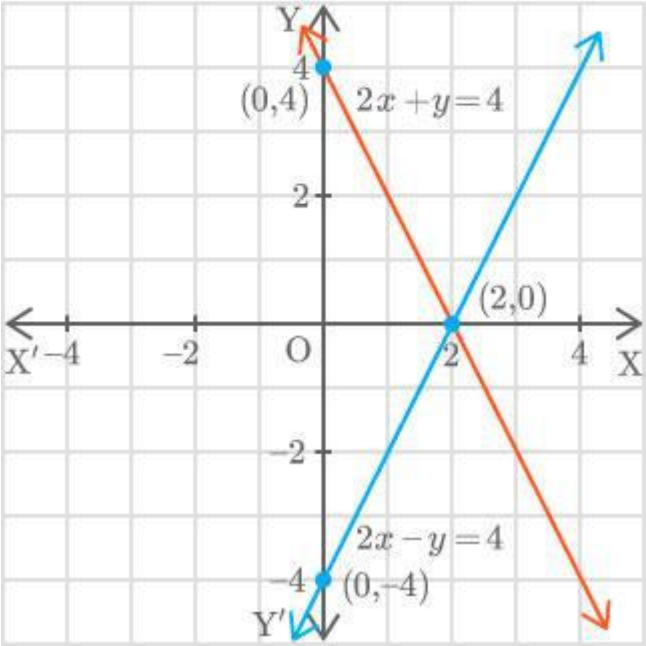
x	0	2
y	4	0

For equation 2:

The points are $(0,-4)$ and $(2,0)$.

$y=2x-4$

x	0	2
y	-4	0



The coordinates of the triangle are $(0,4),(2,0)$ and $(0,-4)$.

Base $=8$

Height $=2$

Area of the triangle $=\frac{1}{2}\times \text{base}\times \text{height}$

$=\frac{1}{2}\times 8\times 2$

$=8$

Thus, the area of the triangle is 8 square unit.

Question 13

Write an equation of a line passing through the point representing the solution of the pair of linear equations, $x+y=2$ and $2x-y=1$. How many such lines can we find?

Solution:

Given equations:

$x+y=2$ ......(1)

$2x-y=1$ ......(2)

Add equations (1) and (2),

$x+y+2x-y=2+1$

$3x=3$

$x=1$

Put the value of x in equation (1),

$x+y=2$

$1+y=2$

$y=2-1$

$y=1$

Thus, $x=1, y=1.$

The line representing the equation $x+2y=3$ passes through $(1,1)$.

There are infinite such lines that can be possibly drawn.

Question 14

If $x+1$ is a factor of $2x^3+ax^2+2bx+1$, then find the values of a and b given that $2a-3b=4$.

Solution:

Let $p(x)=2x^3+ax^2+2bx+1$

$(x+1)$ is a factor of $p(x)$.

$p(-1)=0$

$2(-1)^3+a(-1)^2+2b(-1)+1=0$

$-2+a-2b+1=0$

$a-2b=1$ ......(1)

$2a-3b=4$ ......(2)

Multiply equation (1) by 2,

$2a-4b=2$ ......(3)

Subtract equation (2) from equation (3),

$2a-4b-(2a-3b)=2-4$

$2a-4b-2a+3b=2-4$

$-b=-2$

$b=2$

Put the value of b in equation (1),

$a-2b=1$

$a-2(2)=1$

$a-4=1$

$a=1+4$

$a=5$

Thus, $a=5, b=2$.

Question 15

The angles of a triangle are x, y and $40°$. The difference between the two angles x and y is $30°$. Find x and y.

Solution:

The angles of a triangle are x, y and $40°$.

By the angle sum property,

$x+y+40°=180°$

$x + y = 180° - 40°$

$x + y = 140°$ ......(1)

$x - y = 30°$ ......(2)

Add equation (1) and (2),

$x + y + x - y = 140 + 30$

$2x = 170$

$x = 85$

Put the value of x in equation (2),

$x - y = 30$

$85 - y = 30$

$-y = 30 - 85$

$-y = -55$

$y = 55$

Thus, $x = 85°$, $y = 55°$.

Question 16

Two years ago, Salim was thrice as old as his daughter and six years later, he will be four years older than twice her age. How old are they now?

Solution:

Let Salim's age $= x$ years

Let his daughter's age $= y$ years

2 years ago,

$x - 2 = 3(y - 2)$

$x - 2 = 3y - 6$

$x - 3y = -6 + 2$

$x-3y=-4 \quad ......(1)$

6 years later,

$x+6=2(y+6)+4$

$x+6=2y+12+4$

$x+6=2y+16\ x-2y=16-6$

$x-2y=10 \quad ......(2)$

Subtract equation (1) from equation (2),

$x-2y-(x-3y)=10-(-4)$

$x-2y-x+3y=10+4$

$y=14$

Put the value of y in equation (1),

$x-3(14)=-4$

$x-42=-4$

$x=-4+42$

$x=38$

Thus, the age of Salim is 38 years and his daughter's age is 14 years.

Question 17

The age of the father is twice the sum of the ages of his two children. After 20 years, his age will be equal to the sum of the ages of his children. Find the age of the father.

Solution:

Let the age of father be x years and the sum of the ages of his two children be y years.

$x=2y$

$y = \frac{x}{2}$ ......(1)

After 20 years,

$x + 20 = y + 40$

$x + 20 = \frac{x}{2} + 40$

$2x + 40 = x + 80$

$2x - x = 80 - 40$

Hence, the age of the father is 40 years.

Question 18

Two numbers are in the ratio $5:6$. If 8 is subtracted from each of the numbers, the ratio becomes $4:5$. Find the numbers.

Solution:

Let x and y be two numbers such that

$\frac{x}{y} = \frac{5}{6}$

$6x - 5y = 0$ ......(1)

8 is subtracted from both the numbers, the ratio is $4:5$.

$\frac{x-8}{y-8} = \frac{4}{5}$

$5(x-8) = 4(y-8)$

$5x - 40 = 4y - 32$

$5x - 4y = -32 + 40$

$5x - 4y = 8$ ......(2)

Multiply equation (1) by 4 and equation (2) by 5,

$$24x - 20y = 0 \qquad \text{......}(3)$$

$$25x - 20y = 40 \qquad \text{......}(4)$$

Subtract equation (3) from (4),

$$25x - 20y - (24x - 20y) = 40 - 0 \; 25x - 20y - 24x + 20y = 40 - 0$$

$$x = 40$$

Put the value of x in equation (1),

$$6x - 5y = 0$$

$$6 \times 40 - 5y = 0$$

$$240 - 5y = 0$$

$$5y = 240$$

$$y = \frac{240}{5}$$

$$y = 48$$

Thus, the numbers are 40 and 48.

Question 19

There are some students in two examination halls, A and B. To make the number of students equal in each hall, 10 students are sent from A to B. But if 20 students are sent from B to A, the number of students in A becomes double the number of students in B. Find the number of students in the two halls.

Solution:

Let the number of students in hall A $= x$

Let the number of students in hall B $= y$

10 students are sent from A to B,

$x-10=y+10$

$x=y+20 \qquad ......(1)$

20 students are sent from B to A,

$2(y-20)=x+20$

$2y-40=x+20$

$2y=x+60$

$y=\frac{x+60}{2} \qquad ......(2)$

Substitute (2) in (1),

$x=y+20$

$x=\frac{x+60}{2}+20$

$2x=x+60+40$

$2x-x=100$

$x=100$

Put the value of x in equation (1),

$x=y+20$

$100=y+20$

$y=100-20$

$y=80$

Thus, the number of students in hall A is 100 and in hall B is 80.

Question 20

A shopkeeper gives books on rent for reading. She takes a fixed charge for the first two days, and an additional charge for each day thereafter. Latika

paid Rs 22 for a book kept for six days, while Anand paid Rs 16 for a book kept for four days. Find the fixed charges and the charge for each extra day.

Solution:

Let fixed charge $=\text{Rs. } x$

Let charge for per day $=\text{Rs. } y$

So, Latika kept a book for 6 days.

She pays a fixed charge for 2 days and an additional charge for 4 days.

$$x+4y=22 \qquad ......(1)$$

She pays a fixed charge for 2 days and an additional charge for 2 days.

$$x+2y=16 \qquad ......(2)$$

Subtract equation (2) from equation (1),

$$x+4y-(x+2y)=22-16$$

$$x+4y-x-2y=22-16$$

$$2y=6$$

$$y=3$$

Put the value of y in equation (1),

$$x+4y=22$$

$$x+4(3)=22$$

$$x+12=22$$

$$x=22-12$$

$$x=10$$

Thus, the fixed charge for the first two days is Rs. 10 and charge per day is Rs.

3.

Question 21

In a competitive examination, one mark is awarded for every correct answer while $\frac{1}{2}$ mark is deducted for every wrong answer. Jayanti answered 120 questions and got 90 marks. How many questions did she answer correctly?

Solution:

Let the number of correct answers be x and the number of wrong answers be y.

$$x + y = 120$$

$$x = 120 - y \qquad \text{......(1)}$$

$$x - \frac{1}{2}y = 90 \qquad \text{......(2)}$$

Substitute equation (1) in equation (2)

$$x - \frac{1}{2}y = 90$$

$$120 - y - \frac{1}{2}y = 90$$

$$120 - \frac{3}{2}y = 90$$

$$-\frac{3}{2}y = 90 - 120$$

$$-\frac{3}{2}y = -30$$

$$y = 30 \times \frac{2}{3}$$

$$y = 20$$

Put the value of y in equation (2),

$x = 120 - y$

$= 120 - 20$

$= 100$

Thus, the number of correct answers is 100.

Question 22

The angles of a cyclic quadrilateral ABCD are

$\angle A = (6x + 10)°, \angle B = (5x)°, \angle C = (x + y)°, \angle D = (3y - 10)°$

Find x and y, and hence the values of the four angles.

Solution:

Given:

$\angle A = (6x + 10)°, \angle B = (5x)°, \angle C = (x + y)°, \angle D = (3y - 10)°$

The sum of opposite angles of a cyclic quadrilateral is $180°$.

$\angle A + \angle C = 180$

$6x + 10 + x + y = 180$

$7x + y = 180 - 10$

$7x + y = 170 \quad ......(1)$

$\angle B + \angle D = 180$

$5x + 3y - 10 = 180$

$5x + 3y = 180 + 10$

$5x + 3y = 190 \quad ......(2)$

Multiply equation (1) by 3,

$21x + 3y = 510 \quad ......(3)$

Subtract equation (2) from equation (3),

$$21x+3y-(5x+3y)=510-190\ 21x+3y-5x-3y=510-190$$

$$16x=320$$

$$x=20$$

Put the value of x in equation (2),

$$5x+3y=190$$

$$5(20)+3y=190$$

$$3y=190-100$$

$$3y=90$$

$$y=30$$

The four angles are:

$$\angle A=(6x+10)^\circ$$

$$=(6(20)+10)^\circ$$

$$=(120+10)^\circ$$

$$=130^\circ$$

$$\angle B=(5x)^\circ$$

$$=(5\times 20)^\circ$$

$$=100^\circ$$

$$\angle C=(x+y)^\circ$$

$$=(20+30)^\circ$$

$$=50^\circ$$

$$\angle D=(3y-10)^\circ$$

$$=(3(30)-10)^\circ$$

$=(90-10)°$

$=80°$

Thus, $\angle A=130°, \angle B=100°, \angle C=50°, \angle D=80°$.

Exercise 3.4 (13)

Question 1

Graphically, solve the following pair of equations:

$2x+y=6$

$2x-y+2=0$

Find the ratio of the areas of the two triangles formed by the lines representing these equations with the x-axis and the lines with the y-axis.

Solution:

The given equations are:

$2x+y=6$

$2x-y+2=0$

We can rewrite them as

$y=6-2x$ ......(1)

$y=2x+2$ ......(2)

To represent these equations, you must have at least two solutions for each equation.

For equation 1:

The points are $(0,6)$ and $(3,0)$.

$y=6-2x$

x	0	3

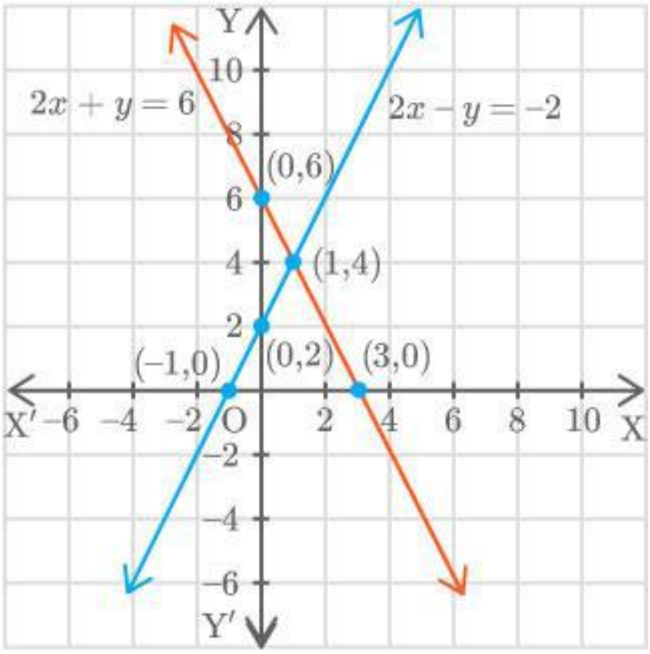
y	6	0

For equation 2:

The points are $(0,2)$ and $(-1,0)$.

$y = 2x + 2$

x	0	-1
y	2	0



Area of triangle ABC $= \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 4 \times 1$

$= 2$

The area of triangle ABC is 2 square unit.

Area of triangle APQ $= \frac{1}{2} \times \text{base} \times \text{height}$

$$=\frac{1}{2}\times 4\times 4$$

$$=8$$

The area of triangle APQ is 8 square unit.

So, $\frac{\text{Area of } \Delta\text{APQ}}{\text{Area of } \Delta\text{ABC}}=\frac{8}{2}=\frac{4}{1}$.

Thus, the ratio of the areas of two triangles is $4:1$.

Question 2

Determine, graphically, the vertices of the triangle formed by the lines

$y=x$ $\qquad$ $3y=x$ $\qquad$ $x+y=8$

Solution:

The given equations are:

$$y=x \qquad \text{......}(1)$$

$$y=\frac{x}{3} \qquad \text{......}(2)$$

$$x+y=8 \qquad \text{......}(3)$$

For equation 1:

The points are $(2,2)$ and $(4,4)$.

$y=x$

x	2	4
y	2	4

For equation 2:

The points are $(3,1)$ and $(6,2)$.

$y = \frac{x}{3}$

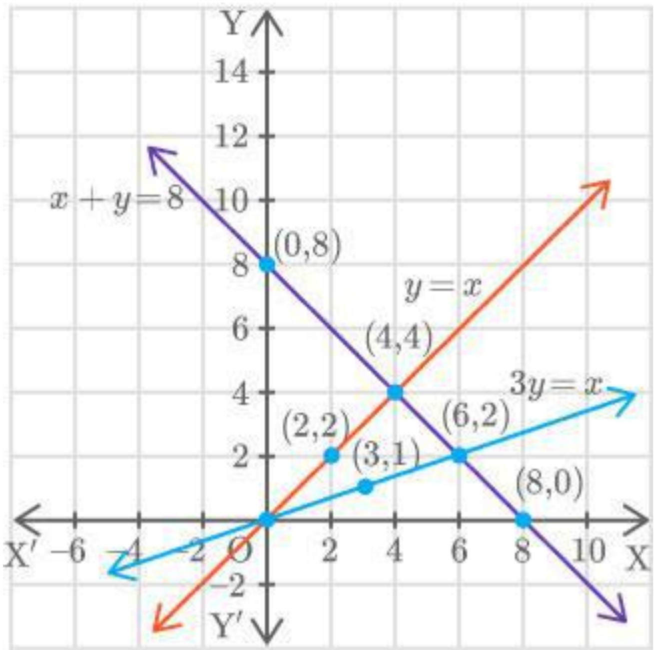
x	3	6
y	1	2

For equation 3:

The points are $(0,8)$ and $(8,0)$.

$y = 8 - x$

x	0	8
y	8	0



The vertices of the triangle are $(0,0), (4,4)$ and $(6,2)$.

❓ Question 3

Draw the graphs for the equations, $x = 3, x = 5$ and $2x - y - 4 = 0$. Also, find the area of the quadrilateral formed by the lines and the x–axis.

Solution:

The given linear equations are:

$x = 3, x = 5$ and $2x - y - 4 = 0$

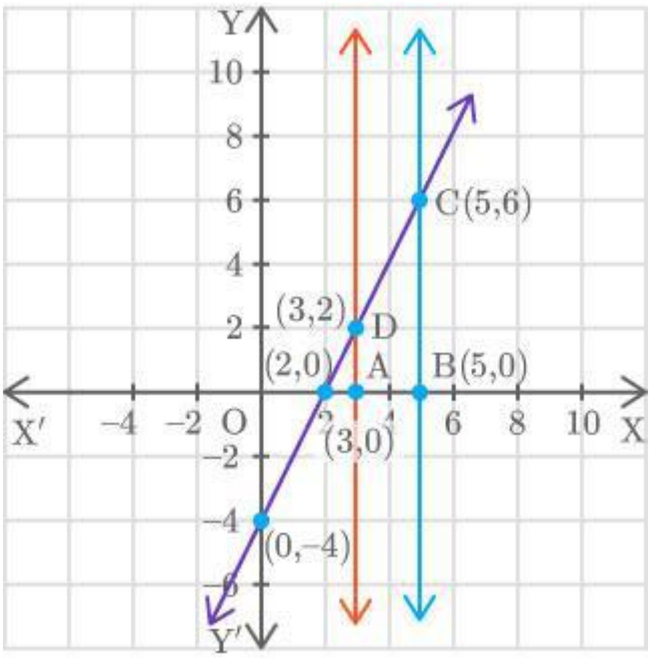
The graph of $x = 3$ is a straight line parallel to y-axis and is at a distance of 3 units right to y-axis.

The graph of $x = 5$ is a straight line parallel to y-axis and is at a distance of 5 units right to y-axis.

$y = 2x - 4$,

The points are $(2,0)$ and $(0,-4)$.

x	2	0
y	0	-4



$AB = 2\ units,\ \ AD = 2\ units$ and $BC = 6\ units$.

Now, calculate the area of the quadrilateral.

$$Required\ area = area\ of\ trapezium\ ABCD$$

$$= \frac{1}{2} \times (sum\ of\ parallel\ sides) \times height = \frac{1}{2}(AD + BC) \times AB$$

$$= \frac{1}{2}(2+6) \times 2$$

$$= 8\ sq.units$$

Thus, the area of the trapezium is 8 sq. units.

Question 4

The cost of 4 pens and 4 pencil boxes is Rs 100. Three times the cost of a pen is Rs 15 more than the cost of a pencil box. Form the pair of linear equations for the above situation. Find the cost of a pen and a pencil box.

Solution:

Let Rs. x be the cost of one pen and Rs. y be the cost of one pencil box.

According to the questions,

$4x + 4y = 100$ ... (1)

$3x = 15 + y$

$3x - y = 15$ ... (2)

Multiply the equation (2) by 4.

$12x - 4y = 60$ ... (3)

Add equation (1) and (3),

$$4x + 4y + 12x - 4y = 100 + 60$$

$$16x = 160$$

$$x = 10$$

Put the value of x in equation (1),

$$4x + 4y = 100$$

$$4(10)+4y=100$$
$$40+4y=100$$
$$4y=100-40$$
$$4y=60$$
$$y=15$$

Thus, the cost of one pen is Rs. 10 and the cost of one pencil box is Rs. 15.

Question 5

Determine algebraically, the vertices of the triangle formed by the lines

$$3x-y=3$$
$$2x-3y=2$$
$$x+2y=8$$

Solution:

The given equations are:

$$3x-y=3$$
$$2x-3y=2$$
$$x+2y=8$$

We can rewrite them as

$$y=3x-3 \qquad ......(1)$$
$$x=\frac{2+3y}{2} \qquad ......(2)$$
$$x=8-2y \qquad ......(3)$$

To represent these equations, you must have at least two solutions for each equation.

For equation 1:

The points are $(0,-3)$ and $(1,0)$.

$y = 3x - 3$

x	0	1
y	-3	0

For equation 2:

The points are $(4,2)$ and $(1,0)$.

$$x = \frac{2+3y}{2}$$

x	4	1
y	2	0

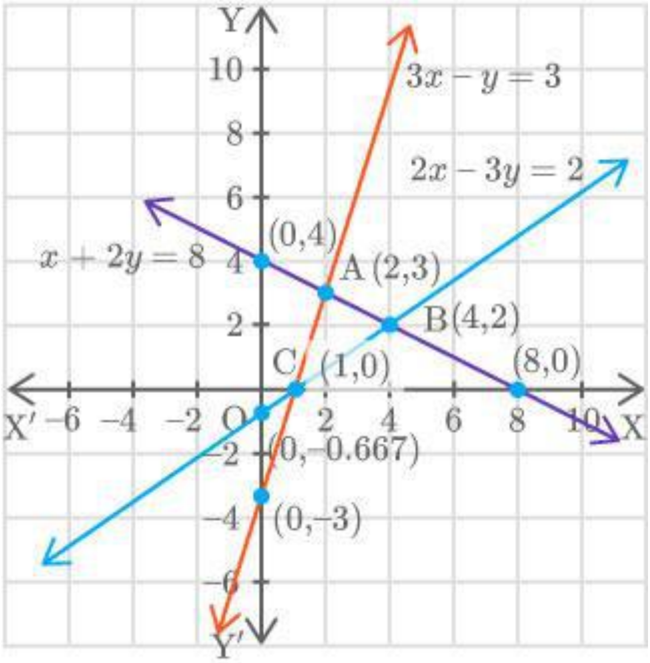
For equation 3:

The points are $(0,4)$ and $(8,0)$.

$x = 8 - 2y$

x	0	8
y	4	0

The graph is:



Thus, the vertices of a triangle are $(2,3),(4,2)$ and $(1,0)$.

❓ Question 6

Ankita travels 14 km to her home partly by rickshaw and partly by bus. She takes half an hour if she travels 2 km by rickshaw and the remaining distance by bus. On the other hand, if she travels 4 km by rickshaw and the remaining distance by bus, she takes 9 minutes longer. Find the speed of the rickshaw and of the bus.

Solution:

Let $x\ \text{km}/\text{h}$ be the speed of the bus and $y\ \text{km}/\text{h}$ be the speed of rickshaw.

Case 1:

Distance covered by rickshaw $= 2\ \text{km}$

Distance covered by bus $= 14\ \text{km} - 2\ \text{km}$

$= 12\ \text{km}$

Time taken by bus $+$ time taken by rickshaw $= \frac{1}{2}$ hours.

$$\frac{12}{x}+\frac{2}{y}=\frac{1}{2} \qquad \text{......(1)}$$

Case 2:

Distance covered by rickshaw $=4\text{ km}$

Distance covered by bus $=14\text{ km}-4\text{ km}$

$=10\text{ km}$

Time taken by bus + time taken by rickshaw $=30\text{ min}+9\text{ min}$

$=39\text{ min}$

$=\frac{39}{60}\text{ hour}.$

$$\frac{10}{x}+\frac{4}{y}=\frac{39}{60} \qquad \text{......(2)}$$

Multiply the equation (1) by 2,

$$\frac{24}{x}+\frac{4}{y}=\frac{2}{2} \qquad \text{......(3)}$$

Subtract equation (3) from equation (2),

$$\frac{10}{x}+\frac{4}{y}-\left(\frac{24}{x}+\frac{4}{y}\right)=\frac{39}{60}-1$$

$$\frac{10}{x}-\frac{24}{x}=\frac{39}{60}-1$$

Take LCM on both sides,

$$\frac{10-24}{x}=\frac{39-60}{60}$$

$$\frac{-14}{x}=\frac{-21}{60}$$

$$x = \frac{60}{21} \times 14$$

$$x = 40\text{ km/h}$$

Put the value of x in equation (1),

$$\frac{12}{x} + \frac{2}{y} = \frac{1}{2}$$

$$\frac{12}{40} + \frac{2}{y} = \frac{1}{2}$$

$$\frac{2}{y} = \frac{1}{2} - \frac{12}{40}$$

$$\frac{2}{y} = \frac{1}{2} - \frac{3}{10}$$

$$\frac{2}{y} = \frac{5-3}{10}$$

$$\frac{2}{y} = \frac{2}{10}$$

$$y = 10\text{ km/h}$$

Thus,

Speed of the bus $= 40\text{ km/h}$

Speed of the rickshaw $= 10\text{ km/h}$

Question 7

A person, rowing at the rate of 5 km/h in still water, takes thrice as much time in going 40 km upstream than in going 40 km downstream. Find the speed of the stream.

Solution:

Let x be the speed of the stream.

Speed of the boat in still water $= 5 \text{ km/h}$

The speed of the boat upstream $= (5 - x) \text{ km/h}$

The speed of the boat downstream $= (5 + x) \text{ km/h}$

Now, according to the condition,

$$\frac{40 \text{ km}}{(5-x) \text{ km/h}} = 3 \times \frac{40 \text{ km}}{(5+x) \text{ km/h}}$$

$$\frac{1}{5-x} = \frac{3}{5+x}$$

$$5 + x = 15 - 3x$$

$$x + 3x = 15 - 5$$

$$4x = 10$$

$$x = \frac{10}{4}$$

$$x = \frac{5}{2} = 2.5 \text{ km/h}$$

Thus, the speed of the stream is 2.5 km/h.

Question 8

A motor boat can travel 30 km upstream and 28 km downstream in 7 hours. It can travel 21 km upstream and return in 5 hours. Find the speed of the boat in still water and the speed of the stream.

Solution:

Let x be the speed of the boat in still water and y be the speed of the stream.

$$\frac{30}{x-y}+\frac{28}{x+y}=7 \qquad \text{......}(1)$$

$$\frac{21}{x-y}+\frac{21}{x+y}=5$$

$$\frac{1}{x-y}+\frac{1}{x+y}=\frac{5}{21}$$

Multiply both sides by 28,

$$\frac{28}{x-y}+\frac{28}{x+y}=\frac{5}{21}\times 28$$

$$\frac{28}{x-y}+\frac{28}{x+y}=\frac{20}{3} \qquad \text{......}(2)$$

Subtract equation (2) from (1),

$$\frac{30}{x-y}+\frac{28}{x+y}-\frac{28}{x-y}-\frac{28}{x+y}=7-\frac{20}{3}\ \frac{30}{x-y}-\frac{28}{x-y}=\frac{21-20}{3}$$

$$\frac{2}{x-y}=\frac{1}{3}$$

$$x-y=6 \qquad \text{......}(3)$$

$$\frac{1}{6}+\frac{1}{x+y}=\frac{5}{21}$$

$$\frac{1}{x+y}=\frac{5}{21}-\frac{1}{6}$$

$$\frac{1}{x+y}=\frac{10-7}{42}$$

$$\frac{1}{x+y}=\frac{3}{42}=\frac{1}{14}$$

$$x+y=14 \qquad \text{......(4)}$$

Add equation (3) and (4),

$$x-y+x+y=6+14$$

$$2x=20$$

$$x=10$$

Put the value of x in equation (3),

$$x-y=6$$

$$10-y=6$$

$$-y=6-10$$

$$-y=-4$$

$$y=4$$

Thus,

Speed of boat in still water $=10\ \text{km}/\text{h}$

Speed of stream $=4\ \text{km}/\text{h}$

Question 9

A two-digit number is obtained by either multiplying the sum of the digits by 8 and then subtracting 5 or by multiplying the difference of the digits by 16 and then adding 3. Find the number.

Solution:

Let the unit digit of the number be x and the ten's digit of the number be y.

The number $=10y+x$

Sum of the digits $=x+y$

$10y+x=8(x+y)-5$

$10y+x=8x+8y-5$

$10y-8y-8x+x=-5$

$2y-7x=-5$

$7x-2y=5 \qquad ......(1)$

The difference of the digits $=y-x \qquad [x<y]$

$10y+x=16(y-x)+3$

$10y+x=16y-16x+3$

$10y-16y+x+16x=3$

$17x-6y=3 \qquad ......(2)$

Multiply equation (1) by 3,

$21x-6y=15 \qquad ......(3)$

Subtract equation (2) from (3),

$21x-6y-(17x-6y)=15-3\ 21x-6y-17x+6y=15-3$

$4x=12\ x=3$

Put the value of x in equation (1),

$7x-2y=5$

$7(3)-2y=5$

$21-2y=5$

$-2y=5-21$

$-2y=-16$

$y=8$

Thus, the unit digit of the number is 3 and the ten's digit number is 8.

The number is 83.

Question 10

A railway half ticket costs half the full fare, but the reservation charges are the same on a half ticket as on a full ticket. One reserved first-class ticket from station A to B costs Rs 2530. Also, one reserved first-class ticket and one reserved first-class half ticket from A to B costs Rs 3810. Find the full first-class fare from station A to B, and also the reservation charges for a ticket.

Solution:

Let x be the cost of a full ticket from station A to B.

Let y be the cost of the reservation.

A reserved first-class ticket costs Rs. 2530

$$x+y=2530 \qquad ......(1)$$

Now, one full and one half first-class reserved ticket cost Rs. 3810.

$$\frac{3}{2}x+2y=3810 \qquad ......(2)$$

Multiply the equation (1) by 2,

$$2x+2y=5060 \qquad ......(3)$$

Subtract the equation (2) from (3),

$$2x+2y-\left(\frac{3}{2}x+2y\right)=5060-3810 \; 2x+2y-\frac{3}{2}x-2y=5060-3810$$

$$\frac{1}{2}x=1250 \; x=2500$$

Put the value of x in equation (1),

$$x+y=2530$$

$2500 + y = 2530$

$y = 2530 - 2500$

$y = 30$

Thus, the first-class ticket from A to B is Rs. 2500 and the reservation cost is Rs. 30.

Question 11

A shopkeeper sells a saree at 8% profit and a sweater at 10% discount, thereby, getting a sum Rs. 1008. If she had sold the saree at 10% profit and the sweater at 8% discount, she would have got Rs. 1028. Find the cost price of the saree and the list price (price before discount) of the sweater.

Solution:

Let x be the CP of the saree and y be the CP of the sweater.

Case 1:

Saree is sold at 8% profit.

$$\text{SP of saree} = x + \frac{8x}{100}$$

$$= \frac{100x + 8x}{100}$$

$$= \frac{108x}{100}$$

The sweater is sold at 10% discount.

$$\text{SP of sweater} = y - \frac{10y}{100}$$

$$= \frac{100y - 10y}{100}$$

$$= \frac{90y}{100}$$

The saree and sweater fetch Rs. 1008.

$$\frac{108x}{100} + \frac{90y}{100} = 1008$$

$$\frac{108x + 90y}{100} = 1008$$

$$108x + 90y = 100800$$

$$6x + 5y = 5600$$

$$x = \frac{5600 - 5y}{6} \qquad \text{......(1)}$$

Saree is sold at 10% profit.

$$\text{SP of saree} = x + \frac{10x}{100}$$

$$= \frac{100x + 10x}{100}$$

$$= \frac{110x}{100}$$

The sweater is sold at 8% discount.

$$\text{SP of sweater} = y - \frac{8y}{100}$$

$$= \frac{100y - 8y}{100}$$

$$= \frac{92y}{100}$$

The saree and sweater fetch Rs. 1028.

$$\frac{110x}{100}+\frac{92y}{100}=1028$$

$$\frac{110x+92y}{100}=1028$$

$$110x+92y=102800 \qquad \ldots\ldots(2)$$ ... (2)

Put the value of x in equation (2),

$$110x+92y=102800$$

$$110\left(\frac{5600-5y}{6}\right)+92y=102800$$

$$\frac{110\times5600}{6}-\frac{110\times5y}{6}+92y=102800$$

$$\frac{616000}{6}-\frac{550y}{6}+92y=102800-\frac{550y}{6}+92y=102800-\frac{616000}{6}$$

Take LCM on both sides,

$$-\frac{550y+552y}{6}=\frac{616800-616000}{6}$$

$$\frac{2y}{6}=\frac{800}{6}$$

$$2y=800$$

$$y=400$$

Put the value of y in equation (1),

$$x=\frac{5600-5y}{6}$$

$$=\frac{5600-5(400)}{6}$$

$$= \frac{5600 - 2000}{6}$$

$$= \frac{3600}{6} = 600$$

Thus, the CP of saree is Rs. 600 and the CP of the sweater is Rs. 400.

Question 12

Susan invested a certain amount of money in two schemes A and B, which offer interest at the rate of 8% per annum and 9% per annum, respectively. She received Rs 1860 as an annual interest. However, had she interchanged the amount of investments in the two schemes, she would have received Rs 20 more as the annual interest. How much money did she invest in each scheme?

Solution:

Let x be the amount invested in scheme A and y be the amount invested in scheme B.

Case 1:

$$8\% \text{ of } x + 9\% \text{ of } y = 1860$$

$$\frac{x \times 1 \times 8}{100} + \frac{y \times 9 \times 1}{100} = 1860$$

$$\frac{8x}{100} + \frac{9y}{100} = 1860$$

$$8x + 9y = 186000 \qquad \ldots\ldots(1)$$

Case 2:

$$9\% \text{ of } x + 8\% \text{ of } y = 1860 + 20 \frac{x \times 1 \times 9}{100} + \frac{y \times 8 \times 1}{100} = 1880$$

$$\frac{9x}{100}+\frac{8y}{100}=1880$$

$$9x+8y=188000 \qquad ......(2)$$

Add equation (1) and (2),

$$8x+9y+9x+8y=186000+188000$$

$$17x+17y=374000$$

$$x+y=22000 \qquad ......(3)$$

Subtract equation (1) from equation (2),

$$9x+8y-(8x+9y)=188000-186000$$

$$9x+8y-8x-9y=2000$$

$$x-y=2000 \qquad ......(4)$$

Add equation (3) and (4),

$$x+y+x-y=22000+2000$$

$$2x=24000$$

$$x=12000$$

Put the value of x in equation (3),

$$x+y=22000$$

$$12000+y=22000$$

$$y=22000-12000$$

$$y=10000$$

Thus, the amount invested in scheme A is Rs. 12000 and in scheme B is Rs. 10000.

Question 13

Vijay had some bananas, and he divided them into two lots A and B. He sold the first lot at the rate of Rs 2 for 3 bananas and the second lot at the rate of

Re 1 per banana and got a total of Rs 400. If he had sold the first lot at the rate of Re 1 per banana, and the second lot at the rate of Rs 4 for 5 bananas, his total collection would have been Rs 460. Find the total number of bananas he had.

Solution:

Let x be the number of bananas in lot A and y be the number of bananas in lot B.

Case 1:

In lot A, the cost of 3 bananas $=\text{Rs. } 2$

In lot A, the cost of x bananas $=\text{Rs. } \frac{2}{3}\times x=\text{Rs. } \frac{2x}{3}$

In lot B, the cost of 1 bananas $=\text{Re. } 1$

In lot B, the cost of y bananas $=\text{Rs. } y$

The total cost of lot A and lot B $=\text{Rs. } 400$

$$\frac{2x}{3}+y=400$$

$$2x+3y=1200 \qquad \ldots\ldots(1)$$

Case 2:

In lot A, the cost of 1 bananas $=\text{Re. } 1$.

In lot A, the cost of x bananas $=\text{Rs. } x$

In lot B, the cost of 5 bananas $=\text{Rs. } 4$.

In lot B, the cost of y bananas $=\text{Rs. } y\times\frac{4}{5}=\text{Rs. } \frac{4y}{5}$

The total cost of lot A and lot B $=\text{Rs. } 460$

$$x+\frac{4y}{5}=460$$

$5x + 4y = 2300 \qquad ......(2)$

Multiply equation (1) by 5 and multiply equation (2) by 2.

$10x + 15y = 6000 \qquad ......(3)$

$10x + 8y = 4600 \qquad ......(4)$

Subtract equation (4) from equation (3),

$10x + 15y - (10x + 8y) = 6000 - 4600$

$10x + 15y - 10x - 8y = 6000 - 4600$

$7y = 1400$

$y = 200$

Put the value of y in equation (1),

$2x + 3y = 1200$

$2x + 3(200) = 1200$

$2x + 600 = 1200$

$2x = 1200 - 600$

$2x = 600$

$x = 300$

The number of bananas in lot A $= 300$

The number of bananas in lot B $= 200$

Total number of bananas $= 300 + 200 = 500$

Thus, the total number of bananas is 500.