NCERT Exemplar

Class 10 Maths

Chapter 2- Polynomials

Exercise 2.1 (Multiple Choice Questions and Answers)

Choose the correct answer from the given four options in the following questions:

Question 1

If one of the zeroes of the quadratic polynomial $(k-1)x^2+kx+1$ is -3, then the value of k is:

(A) $\frac{4}{3}$

(B) $-\frac{4}{3}$

(C) $\frac{2}{3}$

(D) $-\frac{2}{3}$

Solution:

(A)

Let $p(x)=(k-1)x^2+kx+1$

$$p(-3)=0$$

$$(k-1)(-3)^2+k(-3)+1=0$$

$$9(k-1)-3k+1=0$$

$$9k-9-3k+1=0$$

$$6k-8=0$$

$$k = \frac{8}{6}$$

$$k = \frac{4}{3}$$

Question 2

A quadratic polynomial, whose zeroes are -3 and 4, is:

(A) $x^2 - x + 12$

(B) $x^2 + x + 12$

(C) $\frac{x^2}{2} - \frac{x}{2} - 6$

(D) $2x^2 + 2x - 24$

Solution:

(C)

Sum of zeroes $= -3 + 4$

$= 1$

$= \frac{-(-1)}{1}$

$= \frac{-b}{a}$

Product of zeroes $= -3 \times 4$

$= -12$

$$= \frac{-(12)}{1}$$

$$= \frac{c}{a}$$

Now, $a = 1,\ b = -1$ and $c = -12$

Required polynomial

$$= ax^2 + bx + c$$

$$= 1 \cdot x^2 - 1 \cdot x - 12$$

$$= \frac{x^2}{2} - \frac{x}{2} - 6$$

Question 3

If the zeroes of the quadratic polynomial $x^2 + (a+1)x + b$ are 2 and -3, then

(A) $a = -7, b = -1$

(B) $a = 5, b = -1$

(C) $a = 2, b = -6$

(D) $a = 0, b = -6$

Solution:

(D)

Let $p(x) = x^2 + (a+1)x + b$

$$p(2) = 0$$

$(2)^2+(a+1)(2)+b=0$

$4+2a+2+b=0$

$2a+b=-6$

$p(-3)=0$

$(-3)^2+(a+1)(-3)+b=0$

$9-3a-3+b=0$

$3a-b=6$

Now, solve equations $2a+b=-6$ and $3a-b=6$.

$5a=0$

$a=0$

$2\times0+b=-6$

$b=-6$

Thus, $a=0$ and $b=-6$.

Question 4

The number of polynomials having zeroes as -2 and 5 is

(A) 1

(B) 2

(C) 3

(D) more than 3

Solution:

(D)

Sum of zeroes $= -2 + 5$

$= \frac{3}{1}$

$= \frac{-(-3)}{1}$

$= \frac{-b}{a}$

Product of zeroes $= -2 \times 5$

$= \frac{-10}{1}$

$= \frac{c}{a}$

Now, $a = 1, b = -3, c = -10$

Required polynomial

$= ax^2 + bx + c$

$= 1 \cdot x^2 - 3 \cdot x - 10$

$= x^2 - 3x - 10$

When we multiply, or divide any polynomial by any arbitrary constant, the zeroes of the polynomial remain the same.

Multiply by k,

$$p(x) = kx^2 - 3kx - 10k$$

where, k is a real number.

Divide by k,

$$p(x) = \frac{x^2}{k} - \frac{3x}{k} - \frac{10}{k}$$

where, k is a non-zero real number.

Thus, the required number of polynomials is infinite i.e. more than

3.

Question 5

If one of the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$ is zero, the product of the other two zeroes is:

(A) $-\frac{c}{a}$

(B) $\frac{c}{a}$

(C) 0

(D) $-\frac{b}{a}$

Solution:

(B)

Let $p(x) = ax^3 + bx^2 + cx + d$

Let α, β and γ be the zeroes of cubic polynomial $p(x)$, where $\alpha = 0$.

Sum of the product of two zeroes taken at a time $= \frac{c}{a}$

$$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a}$$

$$(0)\beta + \beta\gamma + (0)\gamma = \frac{c}{a}$$

$$\beta\gamma = \frac{c}{a}$$

Thus, the product of the other two zeroes is $\frac{c}{a}$.

Question 6

If one of the zeroes of the cubic polynomial $x^3 + ax^2 + bx + c$ is -1, then the product of the other two zeroes is

(A) $b - a + 1$

(B) $b - a - 1$

(C) $a - b + 1$

(D) $a - b - 1$

Solution:

(A)

Let $p(x) = x^3 + ax^2 + bx + c$

Let α, β and γ be the zeroes of cubic polynomial $p(x)$,

where $\alpha = -1$.

$$p(-1) = 0$$

$$(-1)^3 + a(-1)^2 + b(-1) + c = 0$$

$$-1 + a - b + c = 0$$

$$c = 1 - a + b$$

We know,

$$\text{Product of zeroes} = -\frac{\text{Constant term}}{\text{Coefficient of } x^3}$$

$$= -\frac{c}{1}$$

$$\alpha\beta\gamma = -c$$

$$(-1)\beta\gamma = -c$$

$$\beta\gamma = c$$

$$\beta\gamma = 1 - a + b$$

Thus, the product of the other two zeroes is $1 - a + b$.

Question 7

The zeroes of the quadratic polynomial $x^2 + 99x + 127$ are:

(A) both positive

(B) both negative

(C) one positive and one negative

(D) both equal

Solution:

(B)

Let $p(x) = x^2 + 99x + 127$.

Compare $p(x)$ with $ax^2 + bx + c$.

$a = 1, b = 99, c = 127$

Let α and β be the zeroes of the polynomial $p(x)$.

Sum of zeroes, $(\alpha + \beta) = -\frac{b}{a}$

$= -\frac{99}{1}$

$= -99$

Product of zeroes, $(\alpha\beta) = \frac{c}{a}$

$= \frac{127}{1}$

$= 127$

When the product of zeroes is positive, then either both the zeroes are negative or positive. When the sum of these zeroes is negative, then the zeroes must be negative.

Thus, both zeroes of the given polynomial are negative.

Question 8

The zeroes of the quadratic polynomial $x^2 + kx + k, k \neq 0$

(A) cannot both be positive

(B) cannot both be negative

(C) are always unequal

(D) are always equal

Solution:

(A)

Let $p(x) = x^2 + kx + k, k \neq 0$

Compare $p(x)$ with $ax^2 + bx + c$.

$a = 1, b = k, c = k$

Let α and β be the zeroes of the polynomial $p(x)$.

We know,

Sum of zeroes, $(\alpha + \beta) = -\frac{k}{1}$

$= -k$

Product of zeroes $(\alpha\beta) = \frac{k}{1}$

$= k$

There are two cases.

Case 1: k is negative

When k is negative, $\alpha\beta$ is also negative.

It means α and β are of opposite signs.

Case 2: k is positive

When k is positive, $\alpha\beta$ is also positive, but $\alpha+\beta$ is negative.

When the product of zeroes is positive, then either both the zeroes are negative or both are positive. When the sum of these zeroes is negative, then the zeroes must be negative.

Thus, in any case the zeroes of the given polynomial can't both be positive.

Question 9

If the zeroes of the quadratic polynomial $ax^2+bx+c,\ c\neq 0$ **are equal, then**

(A) c **and** a **have opposite signs**

(B) c **and** b **have opposite signs**

(C) c **and** a **have the same sign**

(D) c **and** b **have the same sign**

Solution:

(C)

Let $p(x)=ax^2+bx+c$

Let α and β be the zeroes of the polynomial $p(x)$.

When $\alpha=\beta$, then they have the same sign (both are positive or both are negative).

For $\alpha\beta > 0$,

Product of zeroes, $(\alpha\beta) = \frac{c}{a}$

So, $\frac{c}{a} > 0$

This is possible only when a and c both have the same sign.

Question 10

If one of the zeroes of a quadratic polynomial of the form $x^2 + ax + b$ is the negative of the other, then it

(A) has no linear term and the constant term is negative.

(B) has no linear term and the constant term is positive

(C) can have a linear term but the constant term is negative.

(D) can have a linear term but the constant term is positive.

Solution:

(A)

Let $p(x) = x^2 + ax + b$

Let α and $-\alpha$ be the zeroes of the given polynomial.

Sum of the zeroes $= \alpha + (-\alpha)$

$= 0$

$= -\frac{a}{1}$

$a = 0$

Now, $p(x) = x^2 + b$, that cannot be linear.

Product of the zeroes $= \alpha(-\alpha) = b$

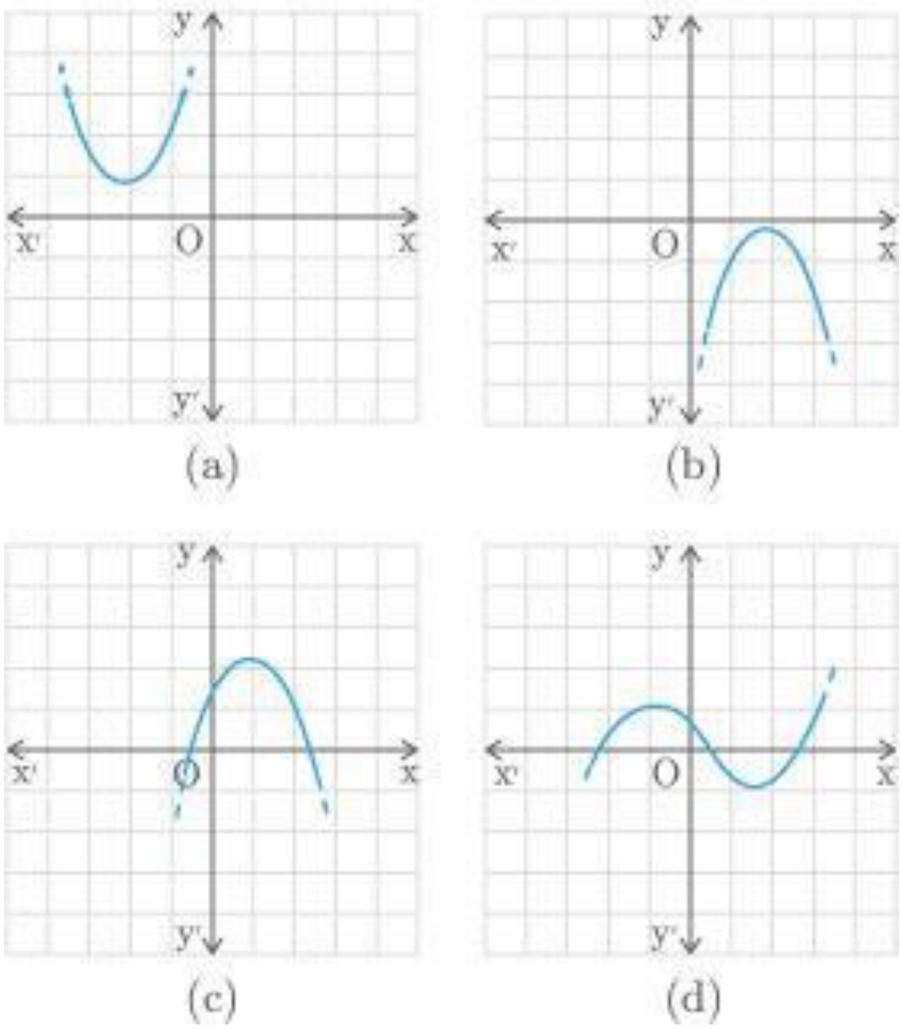
$-\alpha^2 = b$

This is possible when $b < 0$.

Thus, it has no linear term and the constant term is negative.

Question 11

Which of the following is not the graph of a quadratic polynomial?



Solution:

(D)

For any quadratic polynomial, the graph has one of the two shapes; it either opens upwards $\cup$ or opens downwards $\cap$, depending upon whether $a > 0$ or $a < 0$. These curves are known as parabolas.

Thus, option (D) cannot be possible.

Exercise 2.2 (2)

Question 1

Answer the following and justify:

(i) **Can** $x^2 - 1$ **be the quotient on division of** $x^6 + 2x^3 + x - 1$ **by a polynomial in** x **of degree** 5**?**

(ii) **What will the quotient and remainder be on division of** $ax^2 + bx + c$ **by** $px^3 + qx^2 + rx + s,\ p \neq 0$**?**

(iii) **If on division of a polynomial** $p(x)$ **by a polynomial** $g(x)$**, the quotient is zero, what is the relation between the degrees of** $p(x)$ **and** $g(x)$**?**

(iv) **If on division of a non-zero polynomial** $p(x)$ **by a polynomial** $g(x)$**, the remainder is zero, what is the relation between the degrees of** $p(x)$ **and** $g(x)$**?**

(v) **Can the quadratic polynomial** $x^2 + kx + k$ **have equal zeroes for some odd integer** $k > 1$**?**

Solution:

(i) No.

When you divide the polynomial $x^6 + 2x^3 + x - 1$ by a polynomial of degree 2, then the degree of the polynomial of quotient is 4.

By the division algorithm,

Dividend = Divisor × Quotient + Remainder

Degree of divisor + Degree of quotient = Degree of dividend

(ii) Divisor $= px^3 + qx^2 + rx + s, p \neq 0$

Dividend $= ax^2 + bx + c$

When the degree of dividend < the degree of divisor, then the quotient will be zero and remainder is the same as the dividend.

(iii) The relation between the degrees of $p(x)$ and $g(x)$ is:

The degree of $p(x)$ is less than the degree of $g(x)$.

(iv) Thus $g(x)$ is a factor of $p(x)$.

Also, the degree of $g(x)$ is less than or equal to the degree of $p(x)$.

(v) No.

Let $p(x) = x^2 + kx + k$

Let α and α be the zeroes of the polynomial$p(x)$.

We know,

Sum of zeroes $=(\alpha+\alpha)$

$$=-\frac{b}{a}$$

$$2\alpha=-\frac{k}{1}$$

$$=-k$$

$$\alpha=-\frac{k}{2}\text{......................}(1)$$

Product of zeroes $=(\alpha\cdot\alpha)$

$$=\frac{c}{a}$$

$$\alpha^2=\frac{k}{1}$$

$$=k\text{..................................}(2)$$

Now, substitute the value of α from equation (1) into (2).

$$\frac{k^2}{4}=k$$

$$k^2=4k$$

$$k^2-4k=0$$

$$k(k-4)=0$$

$k = 0$ or $k = 4$

But $k > 1$.

Thus, $k = 4$; this is an even, not an odd number.

Question 2

Are the following statements 'True' or 'False'? Justify your answers:

(i) If the zeroes of a quadratic polynomial $ax^2 + bx + c$ are both positive, then a, b and c all have the same sign.

(ii) If the graph of a polynomial intersects the x-axis at only one point, it cannot be a quadratic polynomial.

(iii) If the graph of a polynomial intersects the *x*-axis at exactly two points, it need not be a quadratic polynomial.

(iv) If two of the zeroes of a cubic polynomial are zero, then it does not have linear and constant terms.

(v) If all the zeroes of a cubic polynomial are negative, then all the coefficients and the constant term of the polynomial have the same sign.

(vi) If all three zeroes of a cubic polynomial x^3+ax^2-bx+c are positive, then at least one of a,b and c is non-negative.

(vii) The only value of k for which the quadratic polynomial kx^2+x+k has equal zeros is $\frac{1}{2}$.

Solution:

(i) False.

The zeroes of a quadratic polynomial ax^2+bx+c are both positive then

$$\alpha+\beta=-\frac{b}{a} \text{ and } \alpha\beta=\frac{c}{a}$$

where α and β are zeroes of the quadratic polynomial.

If $c>0$ and $a>0$ then $b<0$

Or $c<0$ and $a<0$ then $b>0$

(ii) False.

A quadratic polynomial may touch the x-axis at exactly one point or intersects x-axis at exactly two points or does not touch the x-axis.

(iii) True.

If the graph of a polynomial intersects the x-axis at exactly two points, then it may or may not be a quadratic polynomial because a polynomial of degree more than 2 is possible that intersects the x-axis at exactly two points.

(iv) True.

Let α, β and γ be the zeroes of the cubic polynomial.

Let $\beta = \gamma = 0$

Also, $p(x) = (x-\alpha)(x-\beta)(x-\gamma)$

$$= (x-\alpha)(x-0)(x-0)$$

$$= x^3 - \alpha x^2$$

This does not have linear and constant terms.

(v) True.

Let α, β and γ be the zeroes.

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

Here, a and b have the same sign.

$$\alpha\beta\gamma = -\frac{d}{a}$$

Here, a, d have the same sign.

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

Here, a, c have the same sign.

So, a, b, c, d have the same sign.

(vi) False.

Let α, β and γ be the three zeroes of the polynomial

$x^3 + ax^2 - bx + c$.

Product of zeroes $(\alpha\beta\gamma) = -\frac{c}{1}$

$$\alpha\beta\gamma = -c$$

All three zeroes are positive. So, the product of all the three zeroes should also be positive.

$$\alpha\beta\gamma > 0$$

$$-c > 0$$

$$c < 0$$

Sum of zeroes $(\alpha + \beta + \gamma) = -\frac{a}{1}$

$$\alpha + \beta + \gamma = -a$$

However, α, β and γ are all positive.

$$\alpha + \beta + \gamma > 0$$

$$-a > 0$$

$$a < 0$$

Sum of the product of two zeroes at a time $= -\frac{b}{1}$

$$\alpha\beta + \beta\gamma + \alpha\gamma = -\frac{b}{1}$$

$$\alpha\beta + \beta\gamma + \alpha\gamma > 0$$

$$-\frac{b}{1} > 0$$

$$b < 0$$

Thus, the cubic polynomial has three positive zeroes when all constants a, b and c are negative.

(vii) False

Let α and α be the two zeroes.

Sum of zeroes $(\alpha + \alpha) = -\frac{b}{a}$

$$2\alpha = -\frac{1}{k}$$

$$\alpha = -\frac{1}{2k} \text{.................}(1)$$

Product of zeroes $(\alpha \cdot \alpha) = \frac{c}{a}$

$$\alpha^2 = \frac{k}{k}$$

$=1$……………………(2)

Solve the equation (1) and (2).

$$\frac{1}{4k^2}=1$$

$$4k^2=1$$

$$k^2=\frac{1}{4}$$

$$k=\pm\frac{1}{2}$$

Now, $k=\frac{1}{2}$ or $k=-\frac{1}{2}$

Thus, for two values of k, the given polynomial has equal zeroes.

Exercise 2.3

Find the zeroes of the following polynomials by factorisation method and verify the relations between the zeroes and the coefficients of the polynomials

Question 1

$4x^2 - 3x - 1$

Solution:

Let $p(x) = 4x^2 - 3x - 1$

$$4x^2 - 3x - 1 = 4x^2 - 4x + x - 1$$

$$= 4x(x-1) + 1(x-1)$$

$$= (x-1)(4x+1)$$

Now,

$x - 1 = 0$ or $4x + 1 = 0$

$x = 1$ or $x = -\frac{1}{4}$

So, the zeroes are 1 and $-\frac{1}{4}$.

Sum of zeroes $= 1 - \frac{1}{4}$

$$=\frac{3}{4}$$

$$=-\frac{(-3)}{4}$$

$$=-\frac{\text{coefficient of x}}{\text{coefficient of x}^2}$$

$$\text{Product of zeroes}=(1)\left(-\frac{1}{4}\right)$$

$$=\frac{-1}{4}$$

$$=\frac{\text{constant term}}{\text{coefficient of x}^2}$$

Thus, the relationship between zeroes and the coefficients of the polynomial is verified.

Question 2

$3x^2+4x-4$

Solution:

Let $p(x)=3x^2+4x-4$

$$3x^2+4x-4=3x^2+6x-2x-4$$

$$= 3x(x+2) - 2(x+2)$$

$$= (x+2)(3x-2)$$

Now,

$$x+2=0 \text{ or } 3x-2=0$$

$$x=-2 \text{ or } x=\frac{2}{3}$$

So, the zeroes are -2 and $\frac{2}{3}$.

Sum of zeroes

$$= -2 + \frac{2}{3}$$

$$= -\frac{4}{3}$$

$$= -\frac{\text{coefficient of x}}{\text{coefficient of } x^2}$$

Product of zeroes

$$= (-2)\left(\frac{2}{3}\right)$$

$$= \frac{-4}{3}$$

$$= \frac{\text{constant term}}{\text{coefficient of } x^2}$$

Thus, the relationship between zeroes and the coefficients of the polynomial is verified.

Question 3

$5t^2 + 12t + 7$

Solution:

Let $p(t) = 5t^2 + 12t + 7$

$$5t^2 + 12t + 7 = 5t^2 + 5t + 7t + 7$$

$$= 5t(t+1) + 7(t+1)$$

$$= (t+1)(5t+7)$$

Now,

$$t + 1 = 0 \text{ or } 5t + 7 = 0$$

$$t = -1 \text{ or } t = -\frac{7}{5}$$

So, the zeroes are -1 and $-\frac{7}{5}$.

Sum of zeroes

$$= -\frac{7}{5} - 1$$

$$= -\frac{12}{5}$$

$$= -\frac{\text{coefficient of t}}{\text{coefficient of t}^2}$$

Product of zeroes

$$= \left(-\frac{7}{5}\right)(-1)$$

$$= \frac{7}{5}$$

$$= \frac{\text{constant term}}{\text{coefficient of t}^2}$$

Thus, the relationship between zeroes and the coefficients of the polynomial is verified.

Question 4

$t^3 - 2t^2 - 15t$

Solution:

Let $p(t) = t^3 - 2t^2 - 15t$

$$t^3 - 2t^2 - 15t = t\left(t^2 - 2t - 15\right)$$

$$= t\left(t^2 - 5t + 3t - 15\right)$$

$$= t\left(t\left(t-5\right) + 3\left(t-5\right)\right)$$

$$= t\left(t-5\right)\left(t+3\right)$$

Now,

$t = 0$ or $t - 5 = 0$ or $t + 3 = 0$

$t = 0$ or $t = 5$ or $t = -3$

So, the zeroes are $0, 5$ and -3.

Sum of zeroes

$$= 0 + 5 + \left(-3\right)$$

$$= 2$$

$$= -\frac{\left(-2\right)}{1}$$

$$= -\frac{\text{coefficient of } t^2}{\text{coefficient of } t^3}$$

Sum of product of zeroes taking two at a time

$$= \left(0\right)\left(5\right) + \left(5\right)\left(-3\right) + \left(-3\right)\left(0\right) \quad = 0 - 15 + 0 = \frac{-15}{1}$$

$$= \frac{\text{coefficient of t}}{\text{coefficient of } t^3}$$

Product of zeroes

$$=(0)(5)(-3)$$

$$=0=\frac{0}{1}=-\left(\frac{0}{1}\right)$$

$$=-\left(\frac{\text{constant term}}{\text{coefficient of } t^3}\right)$$

Thus, the relationship between zeroes and the coefficients of the polynomial is verified.

Question 5

$$2x^2+\frac{7}{2}x+\frac{3}{4}$$

Solution:

Let $p(x)=2x^2+\frac{7}{2}x+\frac{3}{4}$

$$2x^2+\frac{7}{2}x+\frac{3}{4}=\frac{1}{4}\left(8x^2+14x+3\right)$$

$$=\frac{1}{4}\left(8x^2+12x+2x+3\right)$$

$$=\frac{1}{4}\left(4x\left(2x+3\right)+1\left(2x+3\right)\right)$$

$$=\frac{1}{4}\left(2x+3\right)\left(4x+1\right)$$

Now,

$2x+3=0$ or $4x+1=0$

$x=-\frac{3}{2}$ or $x=-\frac{1}{4}$

So, the zeroes are $-\frac{3}{2}$ and $-\frac{1}{4}$.

Sum of zeroes

$$=-\frac{3}{2}-\frac{1}{4}$$

$$=-\frac{7}{4}$$

$$=-\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

Product of zeroes

$$=\left(-\frac{3}{2}\right)\left(-\frac{1}{4}\right)$$

$$=\frac{3}{8}$$

$$= \frac{\text{constant term}}{\text{coefficient of } x^2}$$

Thus, the relationship between zeroes and the coefficients of the polynomial is verified.

Question 6

$4x^2 + 5\sqrt{2}x - 3$

Solution:

Let $p(x) = 4x^2 + 5\sqrt{2}x - 3$

$$4x^2 + 5\sqrt{2}x - 3 = 4x^2 + 6\sqrt{2}x - \sqrt{2}x - 3$$

$$= 2\sqrt{2}x\left(\sqrt{2}x + 3\right) - 1\left(\sqrt{2}x + 3\right)$$

$$= \left(\sqrt{2}x + 3\right)\left(2\sqrt{2}x - 1\right)$$

Now,

$\sqrt{2}x + 3 = 0$ or $2\sqrt{2}x - 1 = 0$

$$x = -\frac{3}{\sqrt{2}} \text{ or } x = \frac{1}{2\sqrt{2}}$$

So, the zeroes are $-\frac{3}{\sqrt{2}}$ and $\frac{1}{2\sqrt{2}}$.

Sum of zeroes

$$= -\frac{3}{\sqrt{2}} + \frac{1}{2\sqrt{2}}$$

$$= -\frac{6+1}{2\sqrt{2}}$$

$$= -\frac{5}{2\sqrt{2}}$$

$$= -\frac{5\sqrt{2}}{4}$$

$$= -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

Product of zeroes

$$= \left(-\frac{3}{\sqrt{2}}\right)\left(\frac{1}{2\sqrt{2}}\right)$$

$$= \frac{-3}{4}$$

$$= \frac{\text{constant term}}{\text{coefficient of } x^2}$$

Thus, the relationship between zeroes and the coefficients of the polynomial is verified.

Question 7

$$2s^2 - \left(1 + 2\sqrt{2}\right)s + \sqrt{2}$$

Solution:

Let $p(s) = 2s^2 - (1 + 2\sqrt{2})s + \sqrt{2}$

$$2s^2 - (1 + 2\sqrt{2})s + \sqrt{2} = 2s^2 - s - 2\sqrt{2}s + \sqrt{2}$$

$$= s(2s - 1) - \sqrt{2}(2s - 1)$$

$$= (2s - 1)(s - \sqrt{2})$$

Now,

$$2s - 1 = 0 \text{ or } s - \sqrt{2} = 0$$

$$s = \frac{1}{2} \text{ or } s = \sqrt{2}$$

So, the zeroes are $\frac{1}{2}$ and $\sqrt{2}$.

Sum of zeroes

$$= \frac{1}{2} + \sqrt{2}$$

$$= \frac{1 + 2\sqrt{2}}{2}$$

$$= -\frac{\left[-(1 + 2\sqrt{2})\right]}{2}$$

$$= -\frac{\text{coefficient of s}}{\text{coefficient of s}^2}$$

Product of zeroes

$$= \left(\frac{1}{2}\right)\left(\sqrt{2}\right)$$

$$= \frac{1}{\sqrt{2}}$$

$$= \frac{\text{constant term}}{\text{coefficient of s}^2}$$

Thus, the relationship between zeroes and the coefficients of the polynomial is verified.

Question 8

$v^2 + 4\sqrt{3}v - 15$

Solution:

Let $p(v) = v^2 + 4\sqrt{3}v - 15$

$$v^2 + 4\sqrt{3}v - 15 = v^2 + 5\sqrt{3}v - \sqrt{3}v - 15$$

$$= v\left(v + 5\sqrt{3}\right) - \sqrt{3}\left(v + 5\sqrt{3}\right)$$

$$= \left(v + 5\sqrt{3}\right)\left(v - \sqrt{3}\right)$$

Now,

$v + 5\sqrt{3} = 0$ or $v - \sqrt{3} = 0$

$v = -5\sqrt{3}$ or $v = \sqrt{3}$

So, the zeroes are $-5\sqrt{3}$ and $\sqrt{3}$.

Sum of zeroes

$= -5\sqrt{3} + \sqrt{3}$

$= -4\sqrt{3}$

$= -\frac{4\sqrt{3}}{1}$

$= -\frac{\text{coefficient of v}}{\text{coefficient of } v^2}$

Product of zeroes

$= \left(-5\sqrt{3}\right)\left(\sqrt{3}\right)$

$= -5 \times 3$

$= -15$

$= \frac{-15}{1}$

$= \frac{\text{constant term}}{\text{coefficient of } v^2}$

Thus, the relationship between zeroes and the coefficients of the polynomial is verified.

Question 9

$y^2+\frac{3}{2}\sqrt{5}y-5$

Solution:

Let $p(y)=y^2+\frac{3}{2}\sqrt{5}y-5$

$$y^2+\frac{3}{2}\sqrt{5}y-5==\frac{1}{2}\left[2y^2+3\sqrt{5}y-10\right]$$

$$=\frac{1}{2}\left[2y^2+4\sqrt{5}y-\sqrt{5}y-10\right]$$

$$=\frac{1}{2}\left[2y\left(y+2\sqrt{5}\right)-\sqrt{5}\left(y+2\sqrt{5}\right)\right]$$

$$=\frac{1}{2}\left[\left(2y-\sqrt{5}\right)\left(y+2\sqrt{5}\right)\right]$$

Now,

$2y-\sqrt{5}=0$ or $y+2\sqrt{5}=0$

$y=\frac{\sqrt{5}}{2}$ or $y=-2\sqrt{5}$

So, the zeroes are $-2\sqrt{5}$ and $\dfrac{\sqrt{5}}{2}$.

Sum of zeroes

$$=-2\sqrt{5}+\frac{\sqrt{5}}{2}$$

$$=-\frac{3\sqrt{5}}{2}$$

$$=-\frac{3}{2}\sqrt{5}$$

$$=-\frac{\text{coefficient of y}}{\text{coefficient of y}^2}$$

Product of zeroes

$$=\left(-2\sqrt{5}\right)\left(\frac{\sqrt{5}}{2}\right)$$

$$=-5$$

$$=\frac{-5}{1}$$

$$=\frac{\text{constant term}}{\text{coefficient of y}^2}$$

Thus, the relationship between zeroes and the coefficients of the polynomial is verified.

Question 10

$$7y^2 - \frac{11}{3}y - \frac{2}{3}$$

Solution:

Let $p(y) = 7y^2 - \frac{11}{3}y - \frac{2}{3}$

$$7y^2 - \frac{11}{3}y - \frac{2}{3} = \frac{1}{3}\left[21y^2 - 11y - 2\right]$$

$$= \frac{1}{3}\left[21y^2 - 14y + 3y - 2\right]$$

$$= \frac{1}{3}\left[7y(3y-2) + 1(3y-2)\right]$$

$$= \frac{1}{3}\left[(3y-2)(7y+1)\right]$$

Now,

$3y - 2 = 0$ or $7y + 1 = 0$

$y = \frac{2}{3}$ or $y = -\frac{1}{7}$

So, the zeroes are $\frac{2}{3}$ and $-\frac{1}{7}$.

Sum of zeroes

$$=\left(\frac{2}{3}\right)+\left(-\frac{1}{7}\right)$$

$$=\frac{11}{21}$$

$$=-\left(-\frac{11}{3\times 7}\right)$$

$$=-\frac{\text{coefficient of y}}{\text{coefficient of y}^2}$$

Product of zeroes

$$=\left(\frac{2}{3}\right)\left(-\frac{1}{7}\right)$$

$$=-\frac{2}{21}$$

$$=-\frac{2}{3\times 7}$$

$$=\frac{-\frac{2}{3}}{7}$$

$$=\frac{\text{constant term}}{\text{coefficient of y}^2}$$

Thus, the relationship between zeroes and the coefficients of the polynomial is verified.

Exercise 2.4

Question 1

For each of the following, find a quadratic polynomial whose sum and product of the zeroes respectively are as given. Also find the zeroes of these polynomials by factorisation.

(i) $\frac{-8}{3}, \frac{4}{3}$

(ii) $\frac{21}{8}, \frac{5}{16}$

(iii) $-2\sqrt{3}, -9$

(iv) $\frac{-3}{2\sqrt{5}}, -\frac{1}{2}$

Solution:

(i) $\frac{-8}{3}, \frac{4}{3}$

Sum of zeroes, $(\alpha+\beta)=\frac{-8}{3}$

Product of zeroes, $(\alpha\beta)=\frac{4}{3}$

Required equation $p(x)=x^2-(\alpha+\beta)x+(\alpha\beta)$

$$=x^2+\frac{8}{3}x+\frac{4}{3}$$

$$=\frac{1}{3}\left[3x^2+8x+4\right] \text{ or } 3x^2+8x+4$$

Factorisation by splitting method,

$$3x^2+8x+4=3x^2+6x+2x+4$$

$$=3x(x+2)+2(x+2)$$

$$=(x+2)(3x+2)$$

Thus, the zeroes are -2 and $-\frac{2}{3}$.

(ii) $\frac{21}{8},\frac{5}{16}$

Sum of zeroes, $(\alpha+\beta)=\frac{21}{8}$

Product of zeroes, $(\alpha\beta)=\frac{5}{16}$

Required equation $p(x)=x^2-(\alpha+\beta)x+(\alpha\beta)$

$$=x^2-\frac{21}{8}x+\frac{5}{16}$$

$$= \frac{1}{16}\left[16x^2 - 42x + 5\right] \text{ or } 16x^2 - 42x + 5$$

Factorisation by splitting method,

$$16x^2 - 42x + 5 = 16x^2 - 40x - 2x + 5$$

$$= 8x(2x - 5) - 1(2x - 5)$$

$$= (2x - 5)(8x - 1)$$

Thus, the zeroes are $\frac{5}{2}$ and $\frac{1}{8}$.

(iii) $-2\sqrt{3}, -9$

Sum of zeroes, $(\alpha + \beta) = -2\sqrt{3}$

Product of zeroes. $(\alpha\beta) = -9$

Required equation $p(x) = x^2 - (\alpha + \beta)x + (\alpha\beta)$

$$= x^2 - \left(-2\sqrt{3}\right)x - 9$$

$$= x^2 + 2\sqrt{3}x - 9$$

Factorisation by splitting method,

$$x^2 + 2\sqrt{3}x - 9 = x^2 + 3\sqrt{3}x - \sqrt{3}x - 9$$

$$= x\left(x + 3\sqrt{3}\right) - \sqrt{3}\left(x + 3\sqrt{3}\right)$$

$$= \left(x + 3\sqrt{3}\right)\left(x - \sqrt{3}\right)$$

Thus, the zeroes are $-3\sqrt{3}$ and $\sqrt{3}$.

(iv) $\frac{-3}{2\sqrt{5}}, -\frac{1}{2}$

Sum of zeroes, $(\alpha+\beta) = \frac{-3}{2\sqrt{5}}$

Product of zeroes, $(\alpha\beta) = -\frac{1}{2}$

Required equation $p(x) = x^2 - (\alpha+\beta)x + (\alpha\beta)$

$= x^2 + \frac{3}{2\sqrt{5}}x - \frac{1}{2}$

$= \frac{1}{2\sqrt{5}}\left[2\sqrt{5}x^2 + 3x - \sqrt{5}\right]$ or $2\sqrt{5}x^2 + 3x - \sqrt{5}$

Factorisation by splitting method,

$2\sqrt{5}x^2 + 3x - \sqrt{5} = 2\sqrt{5}x^2 + 5x - 2x - \sqrt{5}$

$= \sqrt{5}x\left(2x+\sqrt{5}\right) - 1\left(2x+\sqrt{5}\right)$

$= \left(2x+\sqrt{5}\right)\left(\sqrt{5}x - 1\right)$

Thus, the zeroes are $-\frac{\sqrt{5}}{2}$ and $\frac{1}{\sqrt{5}}$.

Question 2

Given that the zeroes of the cubic polynomial $x^3 - 6x^2 + 3x + 10$ are of the form $a, a+b, a+2b$ for some real numbers a and b, find the values of a and b as well as the zeroes of the given polynomial.

Solution:

Let $p(x) = x^3 - 6x^2 + 3x + 10$

Let a, $a+b$ and $a+2b$ are the zeroes of $p(x)$.

$$\text{Sum of the zeroes} = -\frac{(\text{coefficient of } x^2)}{(\text{coefficient of } x^3)}$$

$$a + (a+b) + (a+2b) = -\frac{(-6)}{1}$$

$$3a + 3b = 6$$

$$a + b = 2$$

$$\begin{bmatrix}\text{Sum of the product of} \\ \text{zeroes taking two at a time}\end{bmatrix} = \frac{(\text{coefficient of } x)}{(\text{coefficient of } x^3)}$$

$$a(a+b) + (a+b)(a+2b) + a(a+2b) = \frac{3}{1}$$

$$a(a+b) + (a+b)((a+b)+b) + a((a+b)+b) = \frac{3}{1}$$

Substitute the value of $(a+b)$,

$$a(a+b) + (a+b)((a+b)+b) + a((a+b)+b) = 3$$

$$2a+2(2+b)+a(2+b)=3$$

$$2a+2(2+2-a)+a(2+2-a)=3$$

$$2a+8-2a+4a-a^2=3$$

$$-a^2+8-3+4a=0$$

$$a^2-4a-5=0$$

Factorisation by splitting method,

$$a^2-4a-5=0$$

$$a^2-5a+a-5=0$$

$$a(a-5)+1(a-5)=0$$

$$(a-5)(a+1)=0$$

So, $a=-1$ or 5

When $a=-1$, then $b=3$

When $a=5$, then $b=-3$

The zeroes for $a=-1$ and $b=3$ are

$$a=-1$$

$$a+b=-1+3$$

$$=2$$

$$a+2b=-1+6$$

$$=5$$

The zeroes for $a=5$ and $b=-3$ are

$a = 5$

$a + b = 5 - 3$

$= 2$

$a + 2b = 5 - 6$

$= -1$

Thus, the values of a and b are -1 and 3 or 5 and -3 and the zeroes of the given polynomial are -1, 2 and 5.

Question 3

Given that $\sqrt{2}$ is a zero of the cubic polynomial $6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}$, find its other two zeroes.

Solution:

Let $p(x) = 6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}$

$\sqrt{2}$ is one of the zeroes of $p(x)$.

So, $\left(x - \sqrt{2}\right)$ is one of the factors of given cubic polynomial.

By long division,

$$
\begin{array}{r|l}
 & 6x^2 + 7\sqrt{2}x + 4 \\
\hline
x - \sqrt{2} & 6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2} \\
 & 6x^3 - 6\sqrt{2}x^2 \\
\hline
 & 7\sqrt{2}x^2 - 10x - 4\sqrt{2} \\
 & 7\sqrt{2}x^2 - 14x \\
\hline
 & 4x - 4\sqrt{2} \\
 & 4x - 4\sqrt{2} \\
\hline
 & 0
\end{array}
$$

Fig. 2.4.3

$$6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}$$

$$= \left(6x^2 + 7\sqrt{2}x + 4\right) \times \left(x - \sqrt{2}\right) + 0$$

$$= \left(6x^2 + 4\sqrt{2}x + 3\sqrt{2}x + 4\right) \times \left(x - \sqrt{2}\right)$$

$$= \left(x - \sqrt{2}\right)\left(\sqrt{2}x\left(3\sqrt{2}x + 4\right) + 1\left(3\sqrt{2}x + 4\right)\right)$$

$$= \left(x - \sqrt{2}\right)\left[\left(3\sqrt{2}x + 4\right)\left(\sqrt{2}x + 1\right)\right]$$

$$= \left(x - \sqrt{2}\right)\left(3\sqrt{2}x + 4\right)\left(\sqrt{2}x + 1\right)$$

Thus, the other zeroes are $-\frac{1}{\sqrt{2}}$ and $-\frac{4}{3\sqrt{2}}$

Question 4

Find k so that x^2+2x+k is a factor of $2x^4+x^3-14x^2+5x+6$. Also find all the zeroes of the two polynomials.

Solution:

Apply long division,

$$\begin{array}{r|l} & 2x^2-3x-(2k+8) \\ \hline x^2+2x+k & 2x^4+x^3-14x^2+5x+6 \\ & 2x^4+4x^3+2kx^2 \\ \hline & -3x^3-(2k+14)x^2+5x+6 \\ & -3x^3-6x^2-3kx \\ \hline & -(2k+8)x^2+(5+3k)x+6 \\ & -(2k+8)x^2-(16+4k)x-(2k^2+8k) \\ \hline & (21+7k)x+(2k^2+8k+6) \end{array}$$

Fig. 2.4.4

Since, x^2+2x+k is a factor of the given polynomial, the remainder should be zero.

$7k+21=0$ and $2k^2+8k+6=0$

$k=-3$ and $k^2+4k+3=0$

$k=-3$ and $k^2+3k+k+3=0$

$k=-3$ and $k(k+3)+1(k+3)=0$

$k=-3$ and $(k+3)(k+1)=0$

$k=-3$ and $k=-1$ or $k=-3$

Only $k=-3$ satisfies the required condition.

Now,

Dividend = Divisor × Quotient + Remainder

$$2x^4+x^3-14x^2+5x+6=(x^2+2x-3)(2x^2-3x-2)+0$$

Factorisation by splitting method,

$$(x^2+2x-3)(2x^2-3x-2)$$

$$=(x^2+3x-x-3)(2x^2-4x+x-2)$$

$$=(x(x+3)-1(x+3))(2x(x-2)+1(x-2))$$

$$=(x+3)(x-1)(x-2)(2x+1)$$

Thus, the zeroes of the polynomial x^2+2x-3 are $1,-3$ and

$2x^4+x^3-14x^2+5x+6$ are $1,-3,2,-\frac{1}{2}$.

Question 5

Given that $x-\sqrt{5}$ is a factor of the cubic polynomial $x^3-3\sqrt{5}x^2+13x-3\sqrt{5}$, find all the zeroes of the polynomial.

Solution:

Apply long division,

$$
\begin{array}{r|l}
 & x^2 - 2\sqrt{5}x + 3 \\
\hline
x - \sqrt{5} & x^3 - 3\sqrt{5}x^2 + 13x - 3\sqrt{5} \\
 & x^3 - \sqrt{5}x^2 \\
\hline
 & -2\sqrt{5}x^2 + 13x - 3\sqrt{5} \\
 & -2\sqrt{5}x^2 + 10x \\
\hline
 & 3x - 3\sqrt{5} \\
 & 3x - 3\sqrt{5} \\
\hline
 & 0
\end{array}
$$

Fig. 2.4.5

Now, find the zeroes by division algorithm.

$$x^3 - 3\sqrt{5}x^2 + 13x - 3\sqrt{5} = \left(x^2 - 2\sqrt{5}x + 3\right)\left(x - \sqrt{5}\right) + 0$$

$$= \left(x - \sqrt{5}\right)\left(x^2 - \left\{\left(\sqrt{5} + \sqrt{2}\right) + \left(\sqrt{5} - \sqrt{2}\right)\right\}x + 3\right)$$

$$= \left(x - \sqrt{5}\right)\left(x^2 - \left(\sqrt{5} + \sqrt{2}\right)x - \left(\sqrt{5} - \sqrt{2}\right)x + \left(\sqrt{5} + \sqrt{2}\right)\left(\sqrt{5} - \sqrt{2}\right)\right)$$

$$= \left(x - \sqrt{5}\right)\left(x\left(x - \left(\sqrt{5} + \sqrt{2}\right)\right) - \left(\sqrt{5} - \sqrt{2}\right)\left(x - \left(\sqrt{5} + \sqrt{2}\right)\right)\right)$$

$$= \left(x - \sqrt{5}\right)\left(x - \left(\sqrt{5} + \sqrt{2}\right)\right)\left(x - \left(\sqrt{5} - \sqrt{2}\right)\right)$$

Thus, all the zeroes of the polynomial are $\sqrt{5}, \left(\sqrt{5} + \sqrt{2}\right)$ and $\left(\sqrt{5} - \sqrt{2}\right)$.

Question 6

For which values of a and b, are the zeroes of $q(x)=x^3+2x^2+a$ also the zeroes of the polynomial $p(x)=x^5-x^4-4x^3+3x^2+3x+b$? Which zeroes of p(x) are not the zeroes of $q(x)$?

Solution:

Apply long division,

$$
\begin{array}{r|l}
 & x^2-3x+2 \\
\hline
x^3+2x^2+a & x^5-x^4-4x^3+3x^2+3x+b \\
 & x^5+2x^4+ax^2 \\
\hline
 & -3x^4-4x^3-(a-3)x^2+3x+b \\
 & -3x^4-6x^3-3ax \\
\hline
 & 2x^3-(a-3)x^2+(3+3a)x+b \\
 & 2x^3+4x^2+2a \\
\hline
 & -(a+1)x^2+(3+3a)x+b-2a
\end{array}
$$

Fig. 2.4.6

Here, $q(x)=x^3+2x^2+a$ is a factor of the given polynomial $p(x)=x^5-x^4-4x^3+3x^2+3x+b$. So, the remainder should be zero.

$$-(1+a)x^2+(3+3a)x+(b-2a)=0.x^2+0.x+0$$

Compare the coefficients of x^2 and the constant term.

$$a+1=0$$

$$a = -1$$
$$b - 2a = 0$$
$$b = 2a$$
$$b = 2(-1) = -2$$

Thus, $q(x) = x^3 + 2x^2 - 1$

Now,

$$p(x) = (x^3 + 2x^2 - 1)(x^2 - 3x + 2) + 0$$
$$= (x^3 + 2x^2 - 1)(x^2 - 2x - x + 2)$$
$$= (x^3 + 2x^2 - 1)(x(x-2) - 1(x-2))$$
$$= (x^3 + 2x^2 - 1)(x-2)(x-1)$$

Thus, 1 and 2 are the zeroes of $p(x)$ but not the zeroes of $q(x)$.