NCERT Exemplar

Class 10 Maths

Chapter 1- Real Numbers

Exercise 1.1(10)(Multiple Choice Question and Answers)

Choose the correct answers from the given four options in the following questions:

Question 1

For some integer m, every even integer is of the form

a. m
b. $m+1$
c. $2m$
d. $2m+1$

Solution:

(c)

An integer is a number, without fractional component.

Thus, $m=-2,-1,0,1,2,...$

$2m=-4,-2,0,2,4,...$

It is clear that, every integer of the form $2m$ is even.

Hence, for some integer m, every even integer is of the form $2m$.

Question 2

For some integer q, every odd integer is of the form:

a. q
b. $q+1$
c. $2q$

d. $2q+1$

Solution:

(d)

An integer is a number, without fractional component.

Thus, $q=-2,-1,0,1,2,\ldots$

$2q+1=-3,-1,1,3,5,\ldots$

It is clear, that every integer of the form $2q+1$ is odd.

Hence, for some integer q, every odd integer is of the form $2q+1$.

Question 3

n^2-1 is divisible by 8, if n is:

a. an integer

b. a natural number

c. an odd integer

d. an even integer

Solution:

(c)

Let $p=n^2-1$ ......(1)

There can be two possibilities for n.

Case 1: When n is an even integer.

Let q is an integer such that, $n=2q$

Put $n=2q$ in equation (1)

$p=(2q)^2-1$

$p=4q^2-1$ ......(2)

Put $q=-1$ in equation (2)

$p=4(-1)^2-1$

$p=4-1$

$p=3$

$p=3$ is not divisible by 8.

Now, put $q=0$ in equation (2)

$p=4(0)^2-1$

$p=-1$

$p=-1$ is not divisible by 8.

Case 2: When n is an odd integer.

Let q is an integer such that, $n=2q+1$

Put $n=2q+1$ in equation (1)

$p=(2q+1)^2-1$

$p=4q^2+1+4q-1$

$p=4q^2+4q$

$p=4q(q+1)$ ......(3)

Put $q=-1$ in equation (3)

$p=4(-1)(-1+1)$

$p = 0$

Put $q = 0$ in equation (3)

$p = 4(0)(0+1)$

$p = 0$

Put $q = 1$ in equation (3)

$p = 4 \times 1(1+1)$

$p = 4(2)$

$p = 8$

It is clear that, $p = 0$ and 8 are both divisible by 8.

Hence, $n^2 - 1$ is divisible by 8 when n is an odd integer.

Question 4

If the HCF of 65 and 117 is expressible in the form $65m - 117$, then the value of m is:

- **a. 4**
- **b. 2**
- **c. 1**
- **d. 3**

Solution:

(b)

By Euclid's division algorithm,

$a = bq + r$, where, $0 \le r < b$

Here, a is the dividend, b is the divisor, q is the quotient, and r is the remainder.

By the Euclid's division lemma for 117 and 65.

$117 = 65 \times 1 + 52$

Remainder is $52 \neq 0$.

Now, using division lemma for 65 and 52.

$65 = 52 \times 1 + 13$

Remainder is $13 \neq 0$.

Now, using division lemma for 52 and 13.

$52 = 13 \times 4 + 0$

Remainder is 0 and divisor at the end is 13.

Thus, the HCF of 117 and 65 is 13.

Now, according to the question,

$65m - 117 = 13$

$65m = 13 + 117$

$65m = 130$

$$m = \frac{130}{65}$$

$m = 2$

Hence, the value of m is 2.

Question 5

The largest number which divides 70 and 125, leaving remainders 5 and 8 respectively, is:

a. 13

b. 65

c. 875

d. 1750

Solution:

(a)

The required number leaves remainders 5 and 8 on dividing the numbers 70 and 125.

Thus, the required number is the HCF of $(70-5)$ and $(125-8)$.

Now, find the HCF of 65 and 117.

By Euclid's division algorithm,

$a = bq + r,$ where, $0 \le r < b$

Here, a is the dividend, b is the divisor, q is the quotient, and r is the remainder.

By Euclid's division lemma for 117 and 65.

$117 = 65 \times 1 + 52$

Remainder is $52 \ne 0$.

Now, using division lemma for 65 and 52.

$65 = 52 \times 1 + 13$

Remainder is $13 \ne 0$.

Now, using division lemma for 52 and 13.

$52 = 13 \times 4 + 0$

Remainder is 0 and the divisor at the end is 13.

Thus, the HCF of 117 and 65 is 13.

Hence, the largest number which divides 70 and 125, leaving remainders 5 and 8 respectively is 13.

Question 6

If two positive integers a and b are written as $a = x^3y^2$ and $b = xy^3$, where x, y are prime numbers, then $HCF(a,b)$ is

a. xy

b. xy^2

c. x^3y^3

d. x^2y^2

Solution:

(b)

Given: $a = x^3y^2$ and $b = xy^3$

To find the HCF of a and b.

Factorise $a = x^3y^2$.

$a = (x \times x \times x \times y \times y)$

Factorise $b = xy^3$.

$b = (x \times y \times y \times y)$

Product of least powers of common factors a and b is $(x \times y \times y)$.

Thus, the HCF of a and b is $(x \times y \times y)$.

Hence, $HCF(a,b) = xy^2$.

Question 7

If two positive integers p and q can be expressed as $p=ab^2$ and $q=a^3b$, where a,b being prime numbers, then $\text{LCM}(p,q)$ is equal to

a. ab

b. a^2b^2

c. a^3b^2

d. a^3b^3

Solution:

(c)

Given: $p=ab^2$ and $q=a^3b$

Now, find the LCM of p and q.

Factorise $p=ab^2$.

$p=(a\times b\times b)$

Factorise $q=a^3b$.

$q=(a\times a\times a\times b)$

The product of the highest powers of common prime factors of p and q is $(a\times b\times b\times a\times a)$.

Thus, LCM of p and q is $(a\times b\times b\times a\times a)$.

Hence, $\text{LCM}(p,q)=a^3b^2$.

Question 8

The product of a non-zero rational and an irrational number is

a. **always irrational**
b. **always rational**
c. **rational or irrational**
d. **one**

Solution:

(a)

Consider rational number $\frac{3}{2}$ and irrational number $\sqrt{8}$.

Calculate the product of $\frac{3}{2}$ and $\sqrt{8}$.

$$\frac{3}{2}\times\sqrt{8}=\frac{3}{2}\times 2\sqrt{2}$$
$$=3\sqrt{2}$$

$3\sqrt{2}$ is an irrational number.

Hence, the product of a non-zero rational and an irrational number is always irrational.

Question 9

The least number that is divisible by all the numbers from 1 to 10 (both inclusive)

a. **10**
b. **1000**

c. 504

d. 2520

Solution:

(d)

The least number divisible by all the numbers from 1 to 10 is the LCM of all the numbers from 1 to 10.

Calculate $\text{LCM}(1,2,3,4,5,6,7,8,9,10)$.

Factorise the numbers from 1 to 10,

$1=1$

$2=1\times 2$

$3=1\times 3$

$4=1\times 2\times 2$

$5=1\times 5$

$6=1\times 2\times 3$

$7=1\times 7$

$8=1\times 2\times 2\times 2$

$9=1\times 3\times 3$

$10=1\times 2\times 5$

The product of the highest powers of common prime factors is $(1\times 2\times 2\times 2\times 3\times 3\times 5\times 7)$.

Thus, $\text{LCM}(1,2,3,4,5,6,7,8,9,10)=(1\times 2\times 2\times 2\times 3\times 3\times 5\times 7)$

$=2520$

Hence, the least number divisible by all the numbers from 1 to 10 is 2520.

Question 10

The decimal expansion of rational number $\frac{14587}{1250}$ will terminate after

a. one decimal place

b. two decimal places

c. three decimal places

d. four decimal places

Solution:

(d)

Let the rational number $a = \frac{14587}{1250}$.

Factorise the denominator 1250.

$$a = \frac{14587}{2\times5\times5\times5\times5}$$

Now, multiply numerator and denominator by $(2)^3$

$$a = \frac{14587}{2\times5\times5\times5\times5}\times\frac{2^3}{2^3}$$

$$a = \frac{14587\times8}{10\times1000}$$

$$a = \frac{116696}{10000}$$

$a = 11.6696$

Hence, the decimal expansion of rational number $\frac{14587}{1250}$ will terminate after four decimal places.

Exercise 1.2(10)

Question 1

Write whether every positive integer can be of the form $4q+2$, where q is an integer. Justify your answer.

Solution:

No, every positive integer cannot be of the form $4q+2$.

So, by Euclid's division lemma,

$b=aq+r$ Where, $0\leq r<a$

Since, $(\text{dividend}=\text{divisor}\times\text{quotient}+\text{remainder})$

Here, b is any positive integer and $a=4$,

$b=4q+r$

Where, $0\leq r<4$ i.e. $r=0,1,2,3$

So, this must be of the form $4q$, $4q+1$, $4q+2$, $4q+3$.

Question 2

The product of two consecutive positive integers is divisible by '2'. Is this statement true or false? Give reason.

Solution:

Yes, the statement is true.

Let the two consecutive integers be n and $(n+1)$.

So, one number out of these two must be divisible by 2.

Thus, product of the numbers is also divisible by 2.

Let us consider some examples for this,

3×4 is divisible by 2,

11×12 is divisible by 2,

33×34 is divisible by 2 and so on.

Question 3

'The product of three consecutive positive integers is divisible by 6'. Is this statement true or false? Justify your answer.

Solution:

Yes, the statement is true.

Let the three consecutive integers be n, $(n+1)$ and $(n+2)$.

So, one number out of these three must be divisible by 2 and another one must be divisible by 3.

Hence, the product of numbers is divisible by 6.

Let us consider some examples for this,

$3 \times 4 \times 5$ is divisible by 6,

$10 \times 11 \times 12$ is divisible by 6,

$82 \times 83 \times 84$ is divisible by 6 and so on.

Question 4

Write whether the square of any positive integer can be of the form $3m+2$, where m is a natural number. Justify your answer.

Solution:

No, this statement is not true.

By Euclid's division lemma, $b = aq + r$ where, $0 \le r \le a$.

Here, b is any positive integer and $a = 3$, $b = 3q + r$ for $0 \le r \le 2$.

So, any positive integer is of the form $3k$, $3k+1$ or $3k+2$.

Now, $(3k)^2 = 9k^2 = 3m$ [where, $m = 3k^2$]

and $(3k+1)^2 = 9k^2 + 6k + 1$ [$(a+b)^2 = a^2 + 2ab + b^2$]

$= 3(3k^2 + 2k) + 1$

$= 3m + 1$ [where, $m = 3k^2 + 2k$]

Also,

$(3k+2)^2 = 9k^2 + 12k + 4$ [$(a+b)^2 = a^2 + 2ab + b^2$]

$= 9k^2 + 12k + 3 + 1$

$= 3(3k^2 + 4k + 1) + 1$

$= 3m + 1$ [where, $m = 3k^2 + 4k + 1$]

Which is of the form $3m$ and $3m+1$.

Hence, square of any positive number cannot be of the form $3m+2$.

Question 5

A positive integer is of the form $3q+1$, q being a natural number. Can you write its square in any form other than $3m+1$, i.e., $3m$ or $3m+2$ for some integer m? Justify your answer.

Solution:

No, the statement cannot be true.

By Euclid's division lemma, $b = aq + r$ where, $0 \le r \le a$.

Here, b is any positive integer and $a = 3$, $b = 3q + r$ for $0 \le r \le 2$.

So, this must be of the form $3q$, $3q + 1$ or $3q + 2$.

Now, $3q^2 = 9q^2 = 3m$ [where, $m = 3q^2$]

And,

$$(3q+1)^2 = 9q^2 + 6q + 1$$
$$= 3(3q^2 + 2q) + 1$$
$$= 3m + 1 \quad [\text{where, } m = 3q^2 + 2q]$$

Also,

$$(3q+2)^2 = 9q^2 + 12q + 4$$
$$= 9q^2 + 12q + 3 + 1$$
$$= 3(3q^2 + 4q + 1) + 1$$
$$= 3m + 1$$
$$[\text{where, } m = 3q^2 + 4q + 1]$$

Hence, square of a positive integer of the form $3q + 1$ is always in the form $3m + 1$ for some integer m.

Question 6

The numbers 525 and 3000 are both divisible only by 3, 5, 15, 25, and 75. What is HCF $(525, 3000)$? Justify your answer.

Solution:

To calcuted the HCF of 525 and 3000.

By Euclid's division lemma,

$a = bq + r,$ where, $0 \le r < b$

Here, a is the dividend, b is the divisor, q is the quotient, and r is the remainder.

Since, $[\text{dividend} = \text{divisor} \times \text{quotient} + \text{remainder}]$.

Apply, Euclid's division lemma for 3000 and 525.

$3000 = 525 \times 5 + 375$

Remainder $375 \ne 0$.

Now, apply Euclid's division lemma for 525 and 375.

$525 = 375 \times 1 + 150$

Remainder $150 \ne 0$.

Now, apply Euclid's division lemma for 375 and 150.

$375 = 150 \times 2 + 75$

Remainder $75 \ne 0$.

So, again apply Euclid's division lemma for 150 and 75.

$150 = 75 \times 2 + 0$

Remainder $= 0$.

The numbers 3, 5, 15, 25 and 75 divide the numbers 525 and 3000.
It means, these terms are common in both 525 and 3000.

So, the highest common factor among these is 75.

Question 7

Explain why $3 \times 5 \times 7 + 7$ is a composite number.

Solution:

Let us consider, $a = 3\times5\times7+7$.

$$
\begin{aligned}
a &= 3\times5\times7+7 \\
&= 7(3\times5+1) \\
&= 7\times16
\end{aligned}
$$

Since, a has more than two factors (as 1, 7, 16 and a are factors).

So, it is a composite number.

Question 8

Can two numbers have 18 as their HCF and 380 as their LCM? Give reasons.

Solution:

No, it is not possible.

This is because, HCF is always a factor of LCM but here, 18 is not a factor of 380.

Question 9

Without actually performing the long division, find if $\frac{987}{10500}$ will have terminating or non-terminating (repeating) decimal expansion. Give reasons for your answer.

Solution:

Yes, it is a terminating decimal expansion.

If the simplified denominator has factor in the form of $2^m\times5^n$.

So, this is a terminating decimal.

Now, let us simplify the given expression,

$$\frac{987}{10500}=\frac{47}{500}$$

$$=\frac{47}{5^3\times 2^2}\times\frac{2}{2}$$

$$=\frac{94}{5^3\times 2^3}$$

$$=\frac{94}{1000}$$

$$=0.094$$

Hence, 0.094 is a terminating decimal.

Question 10

A rational number in its decimal expansion is 327.7081. What can you say about the prime factors of q, when this number is expressed in the form $\frac{p}{q}$? Give reasons.

Solution:

The given number, 327.7081 is a terminating decimal number.
So, it represents a rational number and its denominator must have the form $2^m\times 5^n$.

Thus, $327.7081=\frac{3277081}{10000}$

$$=\frac{p}{q}$$

Since, $q = 10000$.

Now, find the factors of q.

$q = 10^4$

$= 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5$

$= 2^4 \times 5^4$

Hence, the prime factorization of q contains only the factors 2 and 5.

Exercise 1.3(14)

Question 1

Show that the square of any positive integer is either of the form $4q$ or $4q+1$ for some integer q.

Solution:

Let us consider, a as an arbitrary positive integer.

Then, by Euclid's division algorithm, for the positive integers a and 4, there is an existence of non–negative integers m and r, such that,

$a = 4m + r$, where $0 \le r < 4$

$$a^2 = (4m + r)^2$$

$$= 16m^2 + r^2 + 8mr \qquad \text{......(1)}$$

Where, $0 \le r < 4$

Now, let us consider the following cases:

Case 1:

When $r = 0$, then put the value of r in equation (1),

$$a^2 = 16m^2$$

$$= 4(4m^2)$$

$$= 4q$$

Where, $q = 4m^2$ is an integer.

Case 2:

When $r = 1$, then put the value of r in equation (1),

$$a^2 = 16m^2 + 1 + 8m$$
$$= 4(4m^2 + 2m) + 1$$
$$= 4q + 1$$

Where, $q = 4(4m^2 + 2m)$ is an integer.

Case 3:

When $r = 2$, then put the value of r in equation (1),

$$a^2 = 16m^2 + 4 + 16m$$
$$= 4(4m^2 + 4m + 1)$$
$$= 4q$$

Where, $q = (4m^2 + 4m + 1)$ is an integer.

Case 4:

When $r = 3$, then put the value of r in equation (1),

$$a^2 = 16m^2 + 24m + 8 + 1$$
$$= 4(4m^2 + 6m + 2) + 1$$
$$= 4q + 1$$

Where, $q = (4m^2 + 6m + 2)$ is an integer.

Hence, the square of any positive integer is either of the form $4q$ or $4q + 1$ for some integer q.

Question 2

Show that cube of any positive integer is of the form $4m$, $4m+1$ or $4m+3$, for some integer m.

Solution:

Let us consider, a as an arbitrary positive integer.

Then, by Euclid's division algorithm, for the positive integers a and 4, there is an existence of non-negative integers q and r such that, $a=4q+r$ where $0\le r<4$.

$$a^3=(4q+r)^3$$

$$=64q^3+r^3+12qr^2+48q^2r$$

$$=64q^3+48q^2r+12qr^2+r^3 \qquad ......(1)$$

Where, $0\le r<4$

Now, let us consider the following cases:

Case 1:

When $r=0$, put the value of r in equation (1)

$$a^3=64q^3$$

$$=4(16q^3)$$

$$=4m$$

Where $m=16q^3$ is an integer.

Case 2:

When $r=1$, put the value of r in equation (1).

$$a^3=64q^3+48q^2+12q+1$$

$$= 4\left(16q^3 + 12q^2 + 3q\right) + 1$$

$$= 4m + 1$$

Where $m = \left(16q^3 + 12q^2 + 3q\right)$ is an integer.

Case 3:

When $r = 2$, put the value of r in equation (1).

$$a^3 = 64q^3 + 48q^2(2) + 12q(4) + 8$$

$$= 4\left(16q^3 + 12q^2(2) + 3q(4) + 2\right)$$

$$= 4m$$

Where $16q^3 + 12q^2(2) + 3q(4) + 2$ is an integer.

Case 4:

When $r = 3$, put the value of r in equation (1),

$$a^3 = 64q^3 + 144q^2 + 108q + 27$$

$$= 64q^3 + 144q^2 + 108q + 24 + 3$$

$$= 4\left(16q^3 + 36q^2 + 27q + 6\right) + 3$$

$$= 4m + 3$$

Where $m = 16q^3 + 36q^2 + 27q + 6$ is an integer.

Hence, the cube of any positive integer is of the form $4m$, $4m+1$, or $4m+3$ for some integer m.

Question 3

Show that the square of any positive integer cannot be of the form $5q+2$ or $5q+3$ for any integer q.

Solution:

Let us consider a as an arbitrary positive integer.

Then, by Euclid's divisions algorithm, for the positive integers a and 5, there is an existence of non-negative integers m and r such that, $a=5m+r$ where $0\le r<5$

Take square on both sides,

$$a^2=(5m+r)^2$$

$$=25m^2+r^2+10mr$$

$$=5(5m^2+2mr)+r^2 \quad ......(1)$$

Now, let us consider the following cases:

Case 1:

When $r=0$, put the value of r in equation (1).

$$a^2=5(5m^2)$$

$$=5q$$

Where, $q=5m^2$ is an integer.

Case 2:

When $r=1$, put the value of r in equation (1).

$$a^2=5(5m^2+2m)+1$$

$$=5q+1$$

Where, $q=5m^2+2m$ is an integer.

Case 3:

When $r=2$, put the value of r in equation (1).

$$a^2=5\left(5m^2+4m\right)+4$$

$$=5q+4$$

Where, $q=5m^2+4m$ is an integer.

Case 4:

When $r=3$, put the value of r in equation (1).

$$a^2=5\left(5m^2+6m\right)+9$$

$$=5\left(5m^2+6m\right)+5+4$$

$$=5\left(5m^2+6m+1\right)+4=5q+4$$

Where, $q=5m^2+6m+1$ is an integer.

Case 5:

When $r=4$, put the value of r in equation (1).

$$a^2=5\left(5m^2+8m\right)+16$$

$$=5\left(5m^2+8m\right)+15+1$$

$$=5\left(5m^2+8m+3\right)+1$$

$$=5q+1$$

Where, $q=5m^2+8m+3$ is an integer.

Hence, the square of any positive integer cannot be of the form $5q+2$ or $5q+3$ for any integer q.

Question 4

Show that the square of any positive integer cannot be of the form $6m+2$ or $6m+5$ for any integer m.

Solution:

Let us consider a as an arbitrary positive integer, Then, by Euclid's division algorithm, for the positive integers a and 6, there is an existence of non-negative integers q and r such that,

$a=6q+r$, where $0\le r<6$

$a^2=(6q+r)^2$

$a^2=36q^2+r^2+12qr$

$a^2=6(6q^2+2qr)+r^2$ ......(1)

Where, $0\le r<6$

Now, let us consider the following cases,

Case 1:

When $r=0$, put the value of r in equation (1),

$a^2=6(6q)^2$

$=6m$

Where, $m=6q^2$ is an integer.

Case 2:

When $r=1$, put the value of r in equation (1),

$a^2=6(6q^2+2q)+1$

$a^2=6m+1$

Where, $m = (6q^2 + 2q)$ is an integer.

Case 3:

When $r = 2$, put the value of r in equation (1).

$a^2 = 6(6q^2 + 4q) + 4$

$a^2 = 6m + 4$

Where, $m = (6q^2 + 4q)$ is an integer.

Case 4:

When $r = 3$, put the value of r in equation (1),

$a^2 = 6(6q^2 + 6q) + 9$

$= 6(6q^2 + 6q) + 6 + 3$

$= 6(6q^2 + 6q + 1) + 3$

$= 6m + 3$

Where, $m = (6q^2 + 6q + 1)$ is an integer.

Case 5:

When $r = 4$, put the value of r in equation (1),

$a^2 = 6(6q^2 + 8q) + 16$

$= 6(6q^2 + 8q) + 12 + 4$

$= 6(6q^2 + 8q + 2) + 4$

$= 6m + 4$

Where, $m = (6q^2 + 8q + 2)$ is an integer.

Case 6:

When $r = 5$, put the value of r in equation (1),

$a^2 = 6(6q^2 + 10q) + 25$

$= 6(6q^2 + 10q) + 24 + 1$

$= 6(6q^2 + 10q + 4) + 1$

$= 6m + 1$

Where, $m = (6q^2 + 10q + 4)$ is an integer.

Hence, the square of any positive integer cannot be of the form $6m + 2$ or $6m + 5$ for any integer m.

Question 5

Show that the square of any odd integer is of the form $4m + 1$, for some integer m.

Solution:

By Euclid's division algorithm,

$a = bq + r, 0 \leq r < 4$ ......(1)

Put the value of b in equation (1).

$a = 4q + r$, where $0 \leq r < 4$ i.e $r = 0, 1, 2, 3$

Now, let us put the different values of r in the equation $a = 4q + r$.

Case 1:

If $r = 0$,

Then, $a = 4q$, which is divisible by 2.

Thus, $4q$ is even.

Case 2:

If $r = 1$,

Then, $a = 4q + 1$ ......(2)

Which is not divisible by 2

Case 3:

If $r = 2$,

$a = 4q + 2 = 2(2q + 1)$, which is divisible by 2

Thus, $2(2q + 1)$ is even

Case 4:

If $r = 3$,

Then, $a = 4q + 3$ ......(3)

Which is not divisible by 2

So, for any positive integer q, $4q + 1$ and $4q + 3$ are odd integers.

Take square on both sides of the equation (2)

$a^2 = (4q + 1)^2$

$= 16q^2 + 1 + 8q$

$= 4(4q^2 + 2q) + 1$ $\left[\because (a + b)^2 = a^2 + b^2 + 2ab\right]$

Which is of the form $4m+1$, where $m=\left(4q^2+2q\right)$ is an integer.

Now, take square on both sides of the equation (3)

$$a^2=\left(4q+3\right)^2$$
$$=16q^2+9+24q$$
$$=4\left(4q^2+6q+2\right)+1$$
$$\left[\because \left(a+b\right)^2=a^2+b^2+2ab\right]$$

Which is of the form $4m+1$, where $m=\left(4q^2+6q+2\right)$ is an integer.

Hence, for some integer m, the square of any odd integer is of the form $4m+1$.

Question 6

If n is an odd integer, then show that n^2-1 is divisible by 8.

Solution:

Let us consider, $a=n^2-1$ ......(1)

It is given that, n is an odd integer.

$\therefore n=1,3,5,\ldots$

Now, put the value of n in equation (1).

$$a=\left(1\right)^2-1$$
$$=1-1$$
$$=0$$

Which is divisible by 8.

Now, put the value of n in equation (1)

$a=(3)^2-1$

$=9-1$

$=8$

Which is divisible by 8.

Now, put the value of n in equation (1).

$a=(5)^2-1$

$=25-1$

$=24$

Which is divisible by 8.

Hence, n^2-1 is divisible by 8, when n is an odd integer.

Question 7

Prove that, if x and y are both odd positive integers, then x^2+y^2 is even but not divisible by 4.

Solution:

Let us consider, $x=2m+1$ and $y=2m+3$ where m is a positive integer.

Then, put the values of x and y in the expression x^2+y^2.

$x^2+y^2=(2m+1)^2+(2m+3)^2$

$=4m^2+1+4m+4m^2+9+12m$

$=8m^2+16m+10$

$=2(4m^2+8m+5)$

Which is even.

Now, to find out, if the given expression is divisible by 4.

$2\left(4m^2+8m+5\right)=4\left(2m^2+2m+2\right)+2$, which is not divisible by 4.

Hence, x^2+y^2 is even for every odd positive integer, but not divisible by 4.

Question 8

Use Euclid's division algorithm to find HCF of 441, 567 **and** 693.

Solution:

Let us consider, $a=693$, $b=567$ and $c=441$

Now, by Euclid's division algorithm,

$a=bq+r, 0\le r<b$ ......(1)

$\left[\because \text{dividend}=\text{divisor}\times\text{quotient}+\text{remainder}\right]$

Using Euclid's division lemma for 693 and 567.

$693=567\times1+126$ $\quad[\because r\ne0]$

Remainder $126\ne0$.

Now, using Euclid's division lemma for 567 and 126

$567=126\times4+63$ $\quad[\because r\ne0]$

Remainder $63\ne0$.

Now, using Euclid's division lemma for 126 and 63 .

$126=63\times2+0$ $\quad[\text{Here}, r=0]$

Remainder $=0$.

$\therefore$ HCF of 693 and 567 $=63$.

Now, let us take $c = 441$ and $d = 63$ and put it in the Euclid's division algorithm,

$c = dq + r \quad 0 \le r < d$ ......(2)

$441 = 63 \times 7 + 0$ $[\text{Here}, r = 0]$

Thus, HCF of (693, 567 and 441) = 63.

Question 9

Using Euclid's division algorithm, find the largest number that divides 1251, 9377 and 15628 leaving remainders 1, 2 and 3 respectively.

Solution:

Since, 1, 2 and 3 are the remainders of 1251, 9377 and 15628, respectively.

Now, subtract these remainders from the given numbers.

We get the following numbers,

$1251 - 1 = 1250$,

$9377 - 2 = 9375$ and

$15628 - 3 = 15625$,

which are divisible by the required number.

Now, the required number is equal to the HCF of 1250, 9375 and 15625.

So, by Euclid's division algorithm,

$a = bq + r, 0 \le r < b$ ......(1)

$[\because \text{dividend} = \text{divisor} \times \text{quotient} + \text{remainder}]$

Now, to find the largest number, use Euclid's division lemma for 15625 and 9375.

$15625 = 9375 \times 1 + 6250$ $[\because r \neq 0]$

Remainder $6250 \neq 0$.

Now, use Euclid's division lemma for 9375 and 6250.

$9375 = 6250 \times 1 + 3125$ $[\because r \neq 0]$

Remainder $3125 \neq 0$.

Again, use Euclid's division lemma for 6250 and 3125.

$6250 = 3125 \times 2 + 0$ $[\text{Here}, r = 0]$

Now, the remainder is 0.

$\therefore$ HCF of 15625 and 9375 $= 3125$.

Now, let us take $a = 1250$ and $b = 3125$ and put it in the Euclid's division algorithm,

$a = bq + r, 0 \leq r < b$ ......(2)

Use Euclid's division lemma for 3125 and 1250 .

$3125 = 1250 \times 2 + 625$ $[\because r \neq 0]$

Remainder $625 \neq 0$.

Now, use Euclid's division lemma for 1250 and 625 .

$1250 = 625 \times 2 + 0$ $[\text{Here}, r = 0]$

Remainder $= 0$.

$\therefore$ HCF of (1250, 9375 and 15625) = 625.

Hence, 625 is the largest number which divides 1251, 9377 and 15628 leaving remainders 1, 2 and 3 respectively.

Question 10

Prove that $\sqrt{3}+\sqrt{5}$ is irrational.

Solution:

Let us suppose that the given expression $\sqrt{3}+\sqrt{5}$ is rational.

Now, let $a=\sqrt{3}+\sqrt{5}$, where a is rational.

So, $\sqrt{3}=a-\sqrt{5}$.

Take square on both sides,

$$\left(\sqrt{3}\right)^2=\left(a-\sqrt{5}\right)^2$$

$$3=a^2+5-2a\sqrt{5}$$

$$2a\sqrt{5}=a^2+2$$

$$\sqrt{5}=\frac{a^2+2}{2a}$$

Which is a contradiction.

This is because on the right-hand side is a rational number, while on the left-hand side is $\sqrt{5}$, which is an irrational number.

Hence, $\sqrt{3}+\sqrt{5}$ is irrational.

Question 11

Show that 12^n cannot end with the digit 0 or 5 for any natural number n.

Solution:

If there is any number which ends with the digits 0 or 5, it is always divisible by 5.

So, if the number, 12^n ends with the digit zero, it must be divisible by 5.

This is possible, only if there is a prime number 5, in the prime factorization of 12^n.

Now, factorise the number 12.

$$12 = 2 \times 2 \times 3$$
$$= 2^2 \times 3$$

Thus, $12^n = \left(2^2 \times 3\right)^n = 2^{2n} \times 3^n$.

Since, the factorisation of the number 12^n does not contain 5.

Hence, there is no value of n (Natural number) for which 12^n ends with digit zero of five.

Question 12

On a morning walk, three persons step off together and their steps measure 40 cm, 42 cm, and 45 cm respectively. What is the minimum distance each should walk, so that each cover the same distance in complete steps?

Solution:

The measurement of steps of three persons 40 cm, 42 cm, and 45 cm.

Now, to find the required minimum distance,

first find the LCM of 40 cm, 42 cm, and 45 cm .

$$40 = 2 \times 2 \times 2 \times 5$$

$42 = 2 \times 3 \times 7$ and

$45 = 3 \times 3 \times 5$

LCM of $40, 42$ and $45 = 2 \times 3 \times 5 \times 2 \times 2 \times 3 \times 7$

$$= 30 \times 12 \times 7$$

$$= 210 \times 12$$

$$= 2520$$

Thus, the minimum distance each should walk is 2520 cm. Also, each can cover the same distance in complete steps.

Question 13

Write the denominator of rational number $\frac{257}{5000}$ in the form $2^m \times 5^n$, where m,n are non-negative integers. Hence, write its decimal expansion, without actual division.

Solution:

Denominator of the given rational number $\frac{257}{5000}$ is 5000.

Now, $5000 = 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5$

$$= (2)^3 \times (5)^4$$

which is of the type $2^m \times 5^n$, where $m = 3$ and $n = 4$ are non-negative integers.

So, simplify the rational number $\frac{257}{5000}$.

$$\frac{257}{5000} = \frac{257}{2^3 \times 5^4} \times \frac{2}{2}$$

$$= \frac{514}{2^4 \times 5^4}$$

$$= \frac{514}{(10)^4}$$

$$= \frac{514}{10000}$$

$$= 0.0514$$

Thus, 0.0514 is the required decimal expansion of the rational number $\frac{257}{5000}$ and it is also a terminating decimal number.

Question 14

Prove that $\sqrt{p}+\sqrt{q}$ is irrational, where p and q are primes.

Solution:

Let us consider, $\sqrt{p}+\sqrt{q}$ to be rational.

Again, let $\sqrt{p}+\sqrt{q}=a$, where a is rational.

So, $\sqrt{q}=a-\sqrt{p}$.

Take square on both sides,

$$q=a^2+p-2a\sqrt{p} \qquad \left[\because (a-b)^2=a^2+b^2-2ab\right]$$

Therefore, $\sqrt{p}=\frac{a^2+p-q}{2a}$, which is a contradiction, as on the right-hand side is a rational number, while on the left-hand side, $\sqrt{p}$ is an irrational number, since p and q are prime numbers.

Hence, $\sqrt{p}+\sqrt{q}$ is irrational.

Exercise 1.4(5)

Question 1

Show that the cube of positive integer of the form $6q+r$**, where** q **is an integer and** $r=0,1,2,3,4,5$ **is also of the form** $6m+r$**.**

Solution:

Let us consider, a as an arbitrary positive integer.

Then, by Euclid's division algorithm, for positive integers a and 6, there is an existence of non-negative integers q and r such that,

$$a=6q+r\text{ , where } 0\le r<6 \qquad ......(1)$$

Take cube on both sides of the equation (1),

$$a^3=(6q+r)^3$$

$$=216q^3+r^3+18qr(6q+r)$$

$$=\left(216q^3+108q^2r+18qr^2\right)+r^3 \qquad ......(2)$$

Where, $0\le r<6$

Case 1:

When $r=0$, put the value of r in equation (2),

$$a^3=\left(216q^3\right)$$

$$=6\left(36q^3\right)$$

$$=6m$$

Where, $m=36q^3$ is an integer.

Case 2:

When $r=1$, put the value of r in equation (2),

$$a^3=\left(216q^3+108q^2+18q\right)+1$$

$$a^3=6\left(36q^3+18q^2+3q\right)+1$$

$$a^3=6m+1$$

Where $m=\left(36q^3+18q^2+3q\right)$ is an integer.

Case 3:

When $r=2$, put the value of r in equation (2),

$$a^3=\left(216q^3+216q^2+72q\right)+8$$

$$a^3=\left(216q^3+216q^2+72q+6\right)+2$$

$$a^3=6\left(36q^3+36q^2+12q+1\right)+2$$

$$a^3=6m+2$$

Where $m=36q^3+36q^2+12q+1$ is an integer.

Case 4:

When $r=3$, put the value of r in equation (2),

$$a^3=\left(216q^3+324q^2+162q\right)+27$$

$$a^3=\left(216q^3+324q^2+162q+24\right)+3$$

$$a^3=6\left(36q^3+54q^2+27q+4\right)+3$$

$$a^3=6m+3$$

Where $m = \left(36q^3 + 54q^2 + 27q + 4\right)$ is an integer.

Case 5:

When $r = 4$, put the value of r in equation (2),

$$a^3 = \left(216q^2 + 432q^2 + 288q\right) + 64$$

$$a^3 = 6\left(36q^3 + 72q^2 + 48q\right) + 60 + 4$$

$$a^3 = 6\left(36q^3 + 72q^2 + 48q + 10\right) + 4$$

$$a^3 = 6m + 4$$

Where $m = \left(36q^3 + 72q^2 + 48q + 10\right)$ is an integer.

Case 6:

When $r = 5$, put the value of r in equation (2),

$$a^3 = \left(216q^3 + 540q^2 + 450q\right) + 125$$

$$a^3 = \left(216q^3 + 540q^2 + 450q + 120\right) + 5$$

$$a^3 = 6\left(36q^3 + 90q^2 + 75q + 20\right) + 5$$

$$a^3 = 6m + 5$$

Where $m = \left(36q^3 + 90q^2 + 75q + 20\right)$ is an integer.

Hence, the cube of a positive integer of the form $6q + r$ where, q is an integer and $r = 0, 1, 2, 3, 4, 5$ is also of the forms $6m$, $6m + 1$, $6m + 2$, $6m + 3$, $6m + 4$, and $6m + 5$ i.e. $6m + r$.

Question 2

Prove that one and only one out of n, $(n+2)$ and $(n+4)$ is divisible by 3, where n is any positive integer.

Solution:

Let us consider n, $(n+2)$ and $(n+4)$.

$\therefore (a,b,c)=(n,n+2,n+4)$

Where n is any positive integer i.e. $n=1,2,3...$

At $n=1$; $(a,b,c)=(1,1+2,1+4)=(1,3,5)$

At $n=2$; $(a,b,c)=(2,2+2,2+4)=(2,4,6)$

At $n=3$; $(a,b,c)=(3,3+2,3+4)=(3,5,7)$

At $n=4$; $(a,b,c)=(4,4+2,4+4)=(4,6,8)$

At $n=5$; $(a,b,c)=(5,5+2,5+4)=(5,7,9)$

At $n=6$; $(a,b,c)=(6,6+2,6+4)=(6,8,10)$

At $n=7$; $(a,b,c)=(7,7+2,7+4)=(7,9,11)$

At $n=8$; $(a,b,c)=(8,8+2,8+4)=(8,10,12)$

Thus, in each set (a,b,c) only one number is a multiple of 3.

Hence, only one out of n, $(n+2)$ and $(n+4)$ is divisible by 3, where, n is any positive integer.

Question 3

Prove that one of any three consecutive positive integers must be divisible by 3.

Solution:

Any three consecutive positive integers must be of the form n, $(n+1)$ and $(n+2)$, where n is any natural number, i.e., $n=1,2,3...$

Let us consider $a=n$, $b=n+1$ and $c=n+2$.

$\therefore (a,b,c)=(n,n+1,n+2)$, where $n=1,2,3...$

At $n=1$; $(a,b,c)=(1,1+1,1+2)=(1,2,3)$

At $n=2$; $(a,b,c)=(2,2+1,2+2)=(2,3,4)$

At $n=3$; $(a,b,c)=(3,3+1,3+2)=(3,4,5)$

At $n=4$; $(a,b,c)=(4,4+1,4+2)=(4,5,6)$

At $n=5$; $(a,b,c)=(5,5+1,5+2)=(5,6,7)$

At $n=6$; $(a,b,c)=(6,6+1,6+2)=(6,7,8)$

At $n=7$; $(a,b,c)=(7,7+1,7+2)=(7,8,9)$

At $n=8$; $(a,b,c)=(8,8+1,8+2)=(8,9,10)$

Thus, in each set (a,b,c) only one number is a multiple of 3.

Hence, one out of any three consecutive positive integers must be divisible by 3.

Question 4

For any positive integer n, prove that n^3-n is divisible by 6.

Solution:

Let us consider $a=n^3-n$..

$a=n(n^2-1)$

$a=n(n-1)(n+1)$ $\quad [\because (a^2-b^2)=(a-b)(a+b)]$

$a=(n-1).n(n+1)$

Observe that this is the product of three consecutive positive integers.

The product of three consecutive positive integers is divisible by 2 and 3. So, it must be divisible by 6.

Hence, n^3-n is always divisible by 6, where n is any positive integer.

Question 5

Show that one and only one out of n, $n+4$, $n+8$, $n+12$, and $n+16$ is divisible by 5, where n is any positive integer.

Solution:

Given numbers aren, $n+4$, $n+8$, $n+12$, and $n+16$, where n is any positive integer.

Let us consider, $n=5q+r$, where $0\le r<5$

Now, $n=5q,\ 5q+1,\ 5q+2,\ 5q+3,\ 5q+4$ for any natural number q [by Euclid's division algorithm].

Let us consider the following cases,

Case 1:

When $n=5q$

$n=5q$ is divisible by 5.

$n+4=5q+4$ is not divisible by 5.

$n+8=5q+8$

$=5q+5+3$

$=5(q+1)+3$ is not divisible by 5.

$n+12=5q+12$

$=5q+10+2$

$=5(q+2)+2$ is not divisible by 5.

$n+16=5q+16$

$=5q+15+1$

$=5(q+3)+1$ is not divisible by 5.

Case 2:

When $n=5q+1$

$n=5q+1$ is not divisible by 5.

$n+4=5q+1+4$

$=5q+5$

$=5(q+1)$ is divisible by 5.

$n+8=5q+1+8$

$=5q+5+4$

$=5(q+1)+4$ is not divisible by 5.

$n+12=5q+1+12$

$=5q+10+3$

$=5(q+2)+3$ is not divisible by 5.

$n+16=5q+1+16$

$=5q+15+2$

$=5(q+3)+2$ is not divisible by 5.

Case 3:

When $n = 5q + 2$

$n = 5q + 2$ is not divisible by 5.

$n + 4 = 5q + 2 + 4$

$= 5q + 5 + 1$

$= 5(q + 1) + 1$ is not divisible by 5.

$n + 8 = 5q + 2 + 8$

$= 5q + 10$

$= 5(q + 2)$ is divisible by 5.

$n + 12 = 5q + 2 + 12$

$= 5q + 10 + 4$

$= 5(q + 2) + 4$ is not divisible by 5.

$n + 16 = 5q + 2 + 16$

$= 5q + 15 + 3$

$= 5(q + 3) + 3$ is not divisible by 5.

Case 4:

When $n = 5q + 3$

$n = 5q + 3$ is not divisible by 5.

$n + 4 = 5q + 3 + 4$

$= 5q + 5 + 2$

$= 5(q + 1) + 2$is not divisible by 5.

$n + 8 = 5q + 3 + 8$

$= 5q + 10 + 1$

$= 5(q + 2) + 1$ is not divisible by 5.

$n+12=5q+3+12$

$=5q+15$

$=5(q+3)$ is divisible by 5.

$n+16=5q+3+16$

$=5q+15+4$

$=5(q+3)+4$ is not divisible by 5.

Case 5:

When $n=5q+4$

$n=5q+4$ is not divisible by 5.

$n+4=5q+4+4$

$=5q+5+3$

$=5(q+1)+3$ is not divisible by 5.

$n+8=5q+4+8$

$=5q+10+2$

$=5(q+2)+2$ is not divisible by 5.

$n+12=5q+4+12$

$=5q+15+1$

$=5(q+3)+1$ is not divisible by 5.

$n+16=5q+4+16$

$=5q+20$

$=5(q+4)$ is divisible by 5.

Hence, in each case, one and only one out of n, $n+4$, $n+8$, $n+12$, and $n+16$ is divisible by 5, where n is any positive integer.